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USING SELF-ORGANIZING MAPS AS A DECISION-MAKING TOOL FOR MULTIOBJECTIVE DESIGN IN ENGINEERING

Felipe Campelo*, Frederico G. Guimarães†, Jaime A. Ramírez*

*Departamento de Engenharia Elétrica, Universidade Federal de Minas Gerais
Av. Antônio Carlos 6627, Belo Horizonte, MG, 31720-010, Brazil

†Departamento de Computação, Universidade Federal de Ouro Preto
Campus Universitário - Morro do Cruzeiro, Ouro Preto, MG, 35400-000, Brasil

Emails: fcampelo@ufmg.br, frederico.g.guimaraes@gmail.com, jramirez@ufmg.br

Abstract— This paper discusses the use of self-organizing maps (SOM) for decision-making in multiobjective design problems. Estimates of the Pareto-optimal solutions for a given problem are mapped onto a two-dimensional grid, where the distance between solutions is a measure of their similarity in the parameter space. By comparing the clustered points and the color grade of those points in the maps, the designer is able to visualize the similarity of the solutions in both the parameter and objective spaces, hence identifying redundant points. With the use of this technique, one is able to work with clusters of solutions instead of individual points, which can simplify the decision-making step.

Keywords— Self-organizing maps, multiobjective optimization, decision-making.

Resumo— Este trabalho discute o uso de mapas auto-organizativos (MAO) como ferramenta de auxílio para tomada de decisões em problemas de projeto multiobjetivo. Estimativas das soluções Pareto-ótimas para um determinado problema são mapeadas sobre uma malha bidimensional, no qual a distância entre as soluções representa uma medida da similaridade das mesmas. Através de comparações entre as posições e cores dadas a cada ponto neste mapa é possível obter uma visualização da semelhança destas soluções, tanto em termos de parâmetros de projeto quanto de performance nos diversos objetivos do problema, o que possibilita a detecção de pontos redundantes e o tratamento de conjuntos de soluções ao invés de pontos individuais, simplificando assim a etapa de tomada de decisões em um determinado projeto.

Palavras-chave— Mapas auto-organizativos, otimização multiobjetivo, tomada de decisões.

1 Introduction

In the design of electromagnetic devices it is usual to have many conflicting objectives, such as device size, energy efficiency, or cost. Due to the very nature of such multiobjective problems, there is no single solution that is optimal for all objectives in the vast majority of cases. Instead, multiobjective optimization algorithms usually return a set of nondominated points, which are estimates of the *Pareto-optimal* solutions.

The existence of these nondominated points is due to the absence of *a priori* preferences or priorities among the multiple objectives in the problem, which generates the need of an *a posteriori* decision-making step in order to complete the design process of a given device. However, the decision-making may become a very difficult task, particularly in cases where the problem has many variables or many objectives. For instance, it is common to have many solutions representing small variations of a same basic design. In this context, clustering methods can be useful when dealing with the estimates of the Pareto-optimal solutions in the decision-making step.

A self-organizing map (SOM) (Kohonen, 1995; Haykin, 1999) is a kind of artificial neural network capable of mapping points from a high-dimensional space onto a low-dimensional, typically two-dimensional, representation or feature space. This technique also performs the clustering of similar solutions, using a given similarity metric and a neighborhood function to preserve the topological properties of the input space. These features make the use of SOM for decision-making in optimization problems an interesting choice, since it provides a low-dimensional view of high-dimensional data, thus helping the analysis and decision-making over the estimates returned by multiobjective optimizers. In fact, this tool has already been explored for mono-objective problems (Igarashi, 2005) with promising results.

In this work we present the use of the self-organizing mapping technique as a tool for simplifying the decision-making process in multiobjective optimization problems. This is achieved by clustering similar solutions into a two-dimensional grid. The SOM algorithm is applied to the estimates of the Pareto optimal solutions found by a general-purpose multiobjective optimization algorithm. By combining the information provided by the color maps, which represent the similarity of the various solutions, the designer is able to gain some qualitative understanding about the problem, as well as to identify clusters of redundant solutions. With this approach, it is possible, for example, to choose just one point from a given cluster for a detailed analysis, instead of having to perform the time consuming task of analyzing all solutions. The proposed methodology has the potential of reducing the complexity of the decision-making process, as well as improving it through visualization.

2 Self-Organizing Maps

Self-organizing maps (SOM) are techniques for clustering and data visualization that provide a mapping of points from a high-dimensional space to a two-dimensional grid, preserving topological properties of the input space, that is, similar points are placed in the neighborhood of each other (Igarashi, 2005).

Given a set P composed of m_0 points in an n-dimensional space, with each dimension normalized to the interval $[0, 1]$, the SOM algorithm starts by the generation of an m x m grid, where each node represents an n-dimensional normalized random vector $\vec{b}_k$ (called *weight vector*). For each point $\vec{p}_i$ from the input set P, the *best matching node* (or *winning node*) i_c, i.e., the node containing the vector with the smallest Euclidean distance from (highest similarity to) $\vec{p}_i$ is found. After that, all weight vectors in the neighborhood of i_c are changed according to the following rule:

$$\vec{b}_{k+1} \leftarrow \vec{b}_k + g_t \left(\vec{p}_i - \vec{b}_{i_c} \right), \tag{1}$$

with the decayment term $g_t \in \{0, 1\}$ as defined below. In this work we employ a square neighborhood with side w, centered at the best matching node i_c. At the end of the first iteration, the solutions from P are mapped to the vector grid. The iterative cycle then continues, with P being mapped onto the $\vec{b}_k$ field obtained from the previous iterations in order to refine the mapping. The values for w and g for a given iteration t are calculated as suggested in (Igarashi, 2005):

$$w_t = 1 + 2r\left(1 - t/T\right), \tag{2}$$

$$g_t = \alpha \left(1 - t/T\right) \exp\left(-d_{j,i_c}^2 / 2\sigma_t^2\right), \tag{3}$$

with:

$$\sigma_t = \sqrt{m/M_1} + \left(\sqrt{m/M_2} - \sqrt{m/M_1} \right) t/T, \tag{4}$$

where r is a constant that determines the initial size of the neighborhood, T is the maximum number of iterations, α is a constant for the learning rate, M_1 and M_2 are user-determined constants that influence the evolution of g_t, and d_{j,i_c} is the lateral distance between the best-matching node $\vec{b}_{i_c}$ and an excited node $\vec{b}_j$. After T iterations, the vector field represents a map of the input points onto the two-dimensional grid, where the distance between two points is a measure of their similarity in the original n-dimensional space.

A more detailed explanation of the concepts behind the SOM equations can be found elsewhere in the literature (Haykin, 1999; Kohonen, 1995; Igarashi, 2005) and will not be given here for the sake of brevity.

3 SOM and Multiobjective Optimization Problems

In multiobjective optimization problems, it is important to deal with the information available from both the parameter and objective spaces. The first provides information on the similarities of the solutions in terms of their design characteristics, while the second provides information on the relative performances of different solutions in terms of the various objectives of the problem. Thus, for applying the SOM to the solutions of multiobjective optimization problems, it is necessary to combine the information contained in both spaces. In this work, we generate p different maps, where p is the number of objectives in the problem. With these maps, which are generated using the same grid, the designer can get qualitative understanding of the solutions, as well as to detect redundant optima.

The procedure for using SOM in multiobjective problems is quite simple: initially, the 2-dimensional grid is generated by following the steps explained in Section 2, with P being the set of estimates of the Pareto-optimal set found by any appropriate multiobjective optimization algorithm. This grid is then used for constructing the maps for the objectives. For each node vector of the grid, the closest sample P_c is determined. Then, the normalized value of the i-th objective function at P_c is attributed to the position of $\vec{b}_k$ in the grid. Therefore, if two points are similar in both spaces, they are going to be closely mapped on the two-dimensional grid, and will also be in regions of similar color in each map. By analyzing the similarities among the maps, the designer is then able to select only one solution from a given cluster of redundant points, so reducing the number of candidate solutions to be considered in the decision-making process.

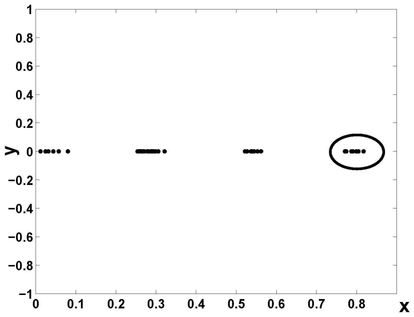
4 Results

4.1 Analytical Problem

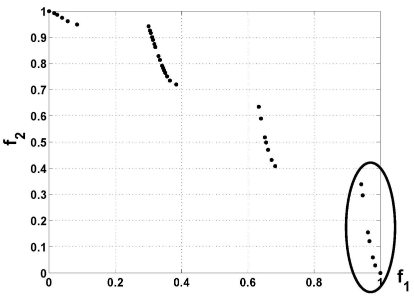
For testing the proposed approach, we start with an analytical problem with two objective functions in the $\mathbb{R}^2$ space, in order to allow for the visual verification of the information provided by the maps on the actual parameter and objective spaces. The problem consists in the minimization of the functions:

$$
\begin{aligned}
f_1(x,y) = &\quad x, \\
f_2(x,y) = &\quad (1+10y)\left[1-\left(\frac{x}{1+10y}\right)^2 - \right. \\
&\left. \left(\frac{x}{1+10y}\right)\sin(8\pi x)\right]
\end{aligned} \tag{5}
$$

with $x, y \in [0,\ 1]$. This problem has been solved in (Guimarães et al., 2007) using the multiobjective clonal selection algorithm (MOCSA). This algorithm was able to sample the Pareto front with 36 nondominated points. The distribution of the solutions over the parameter and objective spaces is shown in Fig. 1.



(a) Space of Parameters



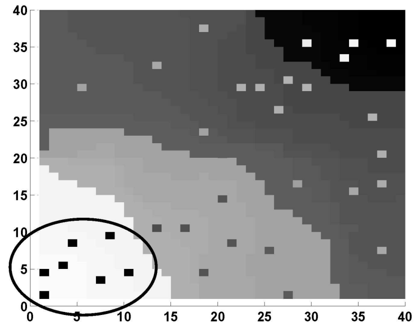
(b) Space of Objectives

Figure 1: Nondominated points in the space of parameters and the space of objectives. Notice that the Pareto front for this problem is discontinuous in both spaces, with gaps along the x axis and the function f_1 axis.

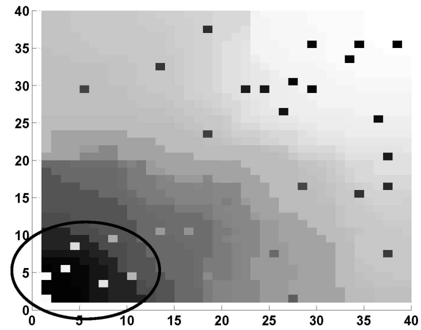
We applied the SOM algorithm over the nondominated points found for this problem, with the following parameters: $m = 40$, $\alpha = 0.6$, $r = 20$, $M_1 = 8$, $M_2 = 25$, and $T = 75$. After the iterative cycle, a map for each objective was generated according to the procedure described in Section 2.

All tests were performed on an AMD Athlon 64 X2 Dual Core (2.20GHz, 3.25GB RAM) PC running Matlab 7.6.0 (R2008a).

Fig. 2 shows the generated maps for this problem. 1.



(a) Map for f_1



(b) Map for f_2

Figure 2: Maps for each objective. It is interesting to see that the map for objective f_1 presents crisp boundaries between the different colors, while the one for f_2 has more smooth transitions. This is due to the gaps on the values of f_1, which can be seen in Fig. 1(b).

By looking at the maps generated, it is possible to extract some qualitative information about the problem. First, it can be easily seen that the color transitions are very abrupt in the map for f_1, and very smooth in the map for f_2. This indicates that the different clusters present very different values for the first objective, but not so much for the second. There are four distinct colored regions in map f_1, which correspond to four main clusters with respect to objective f_1. The smoothness in the color transition in the map for f_2, however, indicates that the performance of the points in this objective does not present big gaps in the objective space. This can be checked by examining Fig.

By focusing, for example, in the points highlighted by black ellipses in Fig. 2, more information about this particular cluster can also be obtained. For instance, observe that the seven points contained in this particular cluster present almost the same color (white) on the f_1 map, and a reasonably smooth range of colors on the f_2 one. From these characteristics we can infer that the first objective function is very stable over this cluster, while the second one presents a smooth variation of values, which is confirmed by an examination of Fig. 1(b).

4.2 Electromagnetic Design

In this section, we investigate the results obtained by applying the SOM technique to the design of a superconducting magnetic energy storage (SMES) device. We have worked on a 3-parameter version of this problem, consisting on the minimization of the three objectives (Guimarães et al., 2006) stated in equations (6) - (8).

$$f_1 = \frac{(B_{stray})^2}{10^{-3}}, \quad (6)$$

$$f_2 = \frac{|Energy - E_{ref}|}{E_{ref}}, \quad (7)$$

$$f_3 = \{|J| + 6.4\,(|B_{max}| - \xi) - 54\}^2\,, \quad (8)$$

Here B_{stray} is the strayed magnetic flux density; $Energy$ is the energy stored by the device; $E_{ref} = 180MJ$ is the desired value for $Energy$; B_{max} and J are the maximum magnetic flux value and the current density at the outer coil of the device; and ξ is a safety parameter defined by the designer. The dimensions of the inner coil are considered constant. For more details on the parameter limits and other characteristics of this problem, see reference (Guimarães et al., 2006).

As we can see in equations (6-8), F_1 accounts for the minimization of the stray field generated by the device; F_2 is a function that represents the deviation of the energy stored by the device from a given reference value; and F_3 aims to produce a solution that is close to the quench limit (in order to avoid the sub-utilization of the superconducting material), but considering a prescribed safety level ξ.

Furthermore, this problem presents a constraint:

$$B_{max} + \frac{J - 54}{6.4} \leq 0, \quad (9)$$

which is very similar to the objective F_3, and is applied for guaranteeing that devices violating the superconducting state will not be allowed in the optimization process.

This problem has been solved using the MOCSA algorithm (Guimarães et al., 2006) with niching in both the parameter and objective spaces, as suggested in (Ávila et al., 2004). At the end of the optimization run, the algorithm returned 197 non-dominated points as estimates of the Pareto set. These points were used for generating the three 100 x 100 maps shown in Fig. 4. The SOM parameters used were $m = 100$, $\alpha = 0.6$, $r = 50$, $M_1 = 8$, $M_2 = 20$, and $T = 75$, as recommended in reference (Igarashi, 2005).

From the generated maps, it is possible to identify a number of clusters of similar points. It is also possible to see that there are no discernible color gaps, i.e., the Pareto set for this problem seems to be reasonably connected.

Let us now consider the two highlighted clusters in Fig. 4. Table 1 shows three points from each cluster. From this table, it is clear that the points in each cluster present a high degree of similarity both in the parameter and the objective spaces, and can therefore be considered redundant.

We can see from the maps that the points in cluster 1 present reasonably good (low) values for the objective f_1, good values for f_2, and a poor performance for objective f_3. Points in cluster 2 present very good performance for f_1 and reasonably good for f_2, and a very poor performance for f_3. It is interesting to notice that the points belonging to cluster 1 present a performance on f_1 almost as good as those in cluster 2, and "win" for objectives 2 and 3. By using this kind of information, the designer can select a given cluster and discard another, based, for example, on subjective criteria or other project requirements not included in the optimization process. In the present case points from cluster 1 seem more attractive than those in cluster 2, unless objective f_1 had a much greater importance than the other two objectives (which is not the case for the problem under consideration). Fig. 3 shows the field contour plot for a sample point from each cluster.

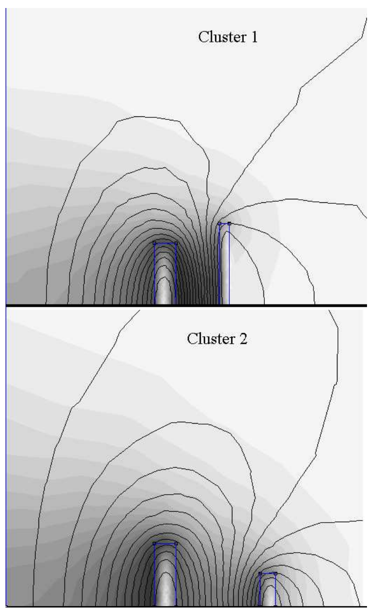


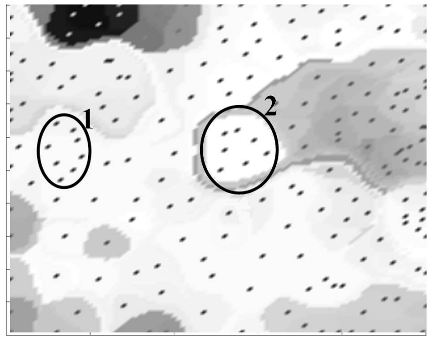
Figure 3: Field plots for sample solutions from the selected clusters

Table 1: Sample Points from the Selected Clusters

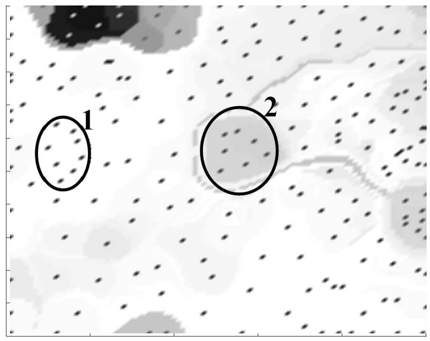
Cluster	r_2	h_2	d_2	F_1	F_2	F_3
1	2.75	2.09	0.12	0.50	0.26	1.75
1	2.72	2.19	0.12	0.40	0.27	1.52
1	2.76	2.06	0.12	0.50	0.25	1.80
2	3.29	0.86	0.19	1.10	0.02	1.17
2	3.30	0.85	0.19	1.10	0.01	1.12
2	3.30	0.80	0.19	1.30	0.01	1.05

5 Conclusion

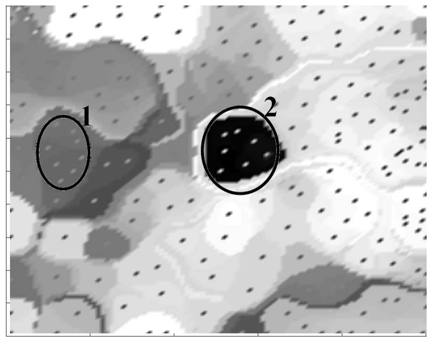
We have explored the use of the self-organizing maps for decision-making in multiobjective design problems. By clearly displaying information contained in the parameter and objective spaces, this technique presents itself as a helpful tool for assisting the designer in the choice of the configuration that will be actually implemented, among the large number of nondominated solutions usually returned by multiobjective optimization algorithms. The SOM can also provide qualitative understanding about the solutions found and can, potentially, be used to analyze the behavior of the objectives in respect to changes in the variables. One limitation of the presented technique is that for problems with a large number of objectives, the analysis of the similarities among the many maps generated can become a cumbersome task. Suggestions for future works include the investigation of the fusion of all objectives into a single map, and the exploration of quantitative metrics for analyzing the detected clusters.



(a) Map for f_1



(b) Map for f_2



(c) Map for f_3

Figure 4: Maps for the three objectives of the SMES problem. In these maps the black represents high values, and white represents low ones. The color of the best matching nodes is inverted for better visualization.

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