Unsupervised and Supervised Techniques for FPSO Electric Power Demand Modelling

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***Abstract*—This work presents a case study of the application of supervised and unsupervised machine learning techniques in the study of three of Petrobras' FPSO units in regards to its equipments' power demand. Specifically, it delves into the outcomes of the clustering and equipment modelling modules of a computational solution developed in a partnership between Universidade Federal Fluminense (UFF) and Petrobras, FPSO Power Demand Analytics (FPDA). The presented results were found satisfactory by UFF and Petrobras' development and engineering teams. For example the equipment modelling methodology resulted in a library of models from which the median absolute error rarely exceeds the 3% mark. The median of the median absolute errors observed across platforms and test scenarios is often less than 1%.**

***Index Terms*—FPSO, Machine Learning, Modelling, Neural Network, Artificial Intelligence, Clustering**

I. Introduction

Floating Production Storage and Offloading (FPSO) units are very common in Brazil's basins [1], boasting the largest global amount, of around 60 units (46 operational) [2]. Following conservative designs, the electrical systems of these platforms historically operate under lower load conditions than initially anticipated during the design phase, allowing for potential production expansion without overload risks: a mere 1% increase in production on a platform with a daily output of 150 thousand barrels [3] at a barrel price of USD 90 (approximately BRL 450) translates to an additional monthly revenue of BRL 20.25 million.

Meanwhile, AI technology is finding important applications in the energy industry, revolutionizing decision-making processes. [4] showcase some noteworthy applications, while [5] highlight machine learning (ML) applications, including predictive models for pipeline criticality assessment [6] and extraction processes [7]. Some other applications include, for example, predicting pressure loss in drilling columns [8], analyzing sedimentary deposits [9], monitoring well drilling processes [10], conducting aerial analysis of on-shore wells [11] and supporting inspection selection [12].

Despite the increasing adoption, the literature on data-driven modeling utilizing historical production data remains limited. Notably, the authors of this study have not encountered any research proposing methodologies for constructing simulators based on production process data from oil platforms. The development of such simulators holds the potential to greatly enhance productivity and generate substantial financial gains.

This paper delves into the outcomes of a computational solution developed alongside Petrobras, FPSO Power Demand Analytics (FPDA), specifically its clustering and equipment modelling modules and their outcomes. FPDA employs machine learning techniques to estimate and simulate load factors of critical electrical equipment on FPSO platforms. The methodology facilitates the evaluation of potential daily production increases for existing FPSOs and aids in designing new FPSOs based on insights gleaned from operational history analysis. Furthermore, the paper explores the adoption of

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clustering techniques to identify different operational modes.

II. FPSO Power Demand Analytics (FPDA)

The FPDA is a computational solution that utilizes machine learning and power flow simulation to analyze and simulate the electrical systems of an FPSO. By employing machine learning models to mimic the behavior of the FPSO's equipment, users can construct the electrical systems of a FPSO unit by integrating these models into an electrical grid. Developed using Python, FPDA takes advantage of libraries such as Scikit-learn [13], PandaPower [14], and Pandas [15].

While machine learning models coupled with power flow simulations were found sufficient to represent a platform's electrical behaviour, it was also desired a clustering functionality, which could give the user the ability to automatically isolate and study certain operational modes. The following sections contextualize the methodology and results observed in the machine learning module and the applied case study with the clustering module.

A. Machine Learning

Figure 1 illustrates the implemented methodology for obtaining the models.

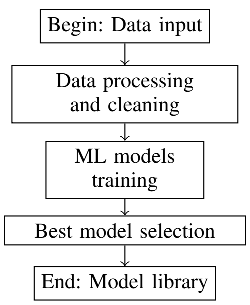

Fig. 1. Detail of the methodology used to construct ML models for each electrical load

After the desired independent variables and respective dependent variable are selected by the user, the corresponding columns are extracted from the dataset. The new dataset is cleaned of any row with missing data (NaNs) and has each variable (column) $c \in C$ converted from the interval $[c_{min}, c_{max}]$ to $[-1, 1]$. The resulting dataset is then split between training (66%) and validation (34%) partitions.

The models that will represent each equipment or system are trained in a two-step grid search, first selecting the best hyperparameter combination of each model type and then selecting the best model based on its validation *Mean Absolute Error*, taken in percentage:

$$\text{MAE\%} = 100 \times \frac{1}{n y_{max}} \sum_{i=1}^{n} |y_{pred_i} - y_{gt_i}| \quad (1)$$

Where y_{max} represents the equipment's maximum power, y_{pred} and y_{gt} represent the prediction and ground truth vectors, respectively, and n represents the cardinality of y_{pred} and y_{gt}: the amount of data points available to assess the models' performance. The best model selected is stored in a model library for later use.

Alternatively, another metric reported is the *Median Absolute Error*, also taken in percentage:

$$\text{MDAE\%} = 100 \times \frac{1}{y_{max}} Med\left(|y_{pred} - y_{gt}|\right) \quad (2)$$

The algorithms developed in this study were trained and analyzed using data collected from three distinct FPSOs, denoted as P1, P2, and P3. For each platform, engineers with knowledge of the specific operations selected the main devices, and their power demand history, synchronized with relevant process variables, was provided. Table I provides a summary of the data available for each platform, including the total number of equipment and process variables, as well as the sampling period of each platform. The equipment predominantly comprises pumping, compression, or injection systems, with compressors arguably being the most important in terms of obtaining accurate predictions.

TABLE I
Time horizon of the training and validation data for each platform

	P1	P2	P3
Sampling period	1 hour	6 minutes	1 hour
Start date	03/23/21	04/13/22	06/05/21
End date	03/23/22	05/10/22	06/05/22
N. devices	35	35	36
N. variables	105	105	109
Total records per var.	8761	6481	8761

To assess the accuracy of trained models, they were tested with data consisting of 1 week (168 data points, one per hour), beginning at the end of the training and validation data's timespan. In other words, the models are tested with data referring to the "next week", which the training algorithm (Fig. 1) has not seen. Table II shows the data details of the scenario constructed to evaluate the models.

TABLE II
Time horizon of each platform's "next week" test data

	Start time	End time
P1	01:00 - 03/23/2022	23:00 - 03/30/2022
P2	01:00 - 05/10/2022	23:00 - 05/17/2022
P3	01:00 - 06/05/2022	23:00 - 06/12/2022

Table III presents the count of equipment that was modeled during the proof of concept phase of the tool. However, due to the data requirements of machine learning techniques, not all models could be trained successfully. In certain cases, the remaining data available for the output variable was observed constant after cleaning (sometimes even *before* cleaning). In other instances, there was missing data throughout the entire duration of the dataset.

Table IV displays the distribution of selected model types for each FPSO, where decision trees, specifically Random Forest Regressions (34 equipments) and Gradient Boosting

TABLE III
AMOUNT OF EQUIPMENTS MODELLED AND ASSESSED IN THE TWO ESTABLISHED SCENARIOS FOR THE CASE STUDY.

	Modelled	Equipments Amount
P1	30	35
P2	23	35
P3	29	36
Total	82	106

Regressions (18 equipments), as well as support vector machines (30 equipments), emerged as the most popular choices, being the most successful. It could be hypothesized that the superior performance of tree-based models could be attributed to their capability to generate discrete values, including zero power demand, unlike models with continuous outputs.

TABLE IV
SUMMARY OF SELECTED MODELS FOR EACH FPSO EQUIPMENT

	P1	P2	P3	Total
Multilayer Perceptron	0	0	0	0
Multivariate Regression	0	0	0	0
Support-Vector Machine	5	10	15	30
Gradient Boosting Regression	10	4	4	18
Random Forest Regression	15	9	10	34
Ridge Regression	0	0	0	0
Decision Tree Regression	0	0	0	0
Total	30	23	29	82

Table V showcases the validation set's mean absolute error in percentage (MAE%) distribution for the equipments of the three platforms. Results show adequate fit, confirming the proposed methodological process.

TABLE V
SUMMARY OF MAE% FOR THE VALIDATION SET

	P1	P2	P3
Avg.	0.5249	0.6497	2.0437
IQR	0.4565	0.7181	1.6840
25%	0.1682	0.1663	0.7329
50%	0.3760	0.3172	1.4963
75%	0.6247	0.8844	2.4169
Max	3.0545	2.5718	9.5857

Model performance is confirmed in Table VI (next week projection), with the third quartile's MAE% at around 10% in the worst case scenario (platform P2): all others are below the 3% mark. Attention is drawn to the clear outlier with a MAE% of 86%, likely due to missing data during training or another correlated issue.

Tables VII and VIII showcase the median absolute error in percentage (MDAE%) for the validation and test sets, respectively. Upon inspection it can be hypothesized that the median errors are generally lower than the mean errors.

TABLE VI
SUMMARY OF MAE% FOR THE TEST SET

	P1	P2	P3
Avg.	1.2423	4.9811	1.9879
IQR	0.8196	1.6427	1.2651
25%	0.0529	0.2386	0.3134
50%	0.4193	0.9398	0.8603
75%	0.8725	1.8813	1.5785
Max	18.7205	85.1536	18.9521

TABLE VII
SUMMARY OF MDAE% FOR THE VALIDATION SET

	P1	P2	P3
Avg.	0.1380	0.2951	0.5493
IQR	0.2123	0.3804	0.8661
25%	0.0005	0.0159	0.0212
50%	0.0629	0.0990	0.5054
75%	0.2128	0.3963	0.8873
Max	0.6399	1.8675	1.6945

B. Clustering

Figure 2 shows the steps performed by the implemented clustering algorithm.

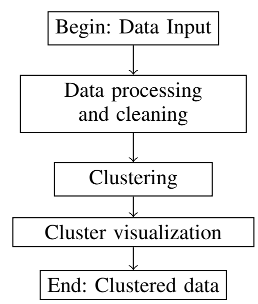

Fig. 2. Implemented algorithm's structure

The tests were performed on the same dataset as the one with which the machine learning models were trained and validated (Table I). The expected datasets to be supplied have a certain percentage of missing data (NaNs), needing thus special preprocessing. A simple strategy was applied, where first, columns with NaNs percentages above 10% are removed; Then, all rows with missing data are removed, followed by columns with constant values. Finally, all columns $c \in C$ are converted from the interval $[c_{min}, c_{max}]$ to $[-1, 1]$.

Four clustering techniques were implemented and tested:

- **K-Means**: The algorithm known as K-Means [16] is one of the most tradicional clustering algorithms used, splitting samples into groups (clusters) with the objective of minimizing the sum of squares of the norms between the clusters' samples and centroid;
- **Mean-Shift**: The Mean-Shift strategy [17] utilizes a Hill Climbing [18] algorithm that iteratively searches for local maximums in the density of samples;

TABLE VIII
SUMMARY OF MDAE% FOR THE TEST SET

	P1	P2	P3
Avg.	0.2893	5.2201	0.7354
IQR	0.3581	1.4583	0.7196
25%	0.0051	0.1950	0.2154
50%	0.1380	0.7015	0.4965
75%	0.3632	1.6533	0.9350
Max	2.0394	94.1706	3.5306

- **DBSCAN**: The DBSCAN algorithm [19] works by separating high density areas (clusters) from low density areas (not-clusters). Appropriate criteria for establishing a relationship between a sample and a cluster are defined through input parameters that tune the behavior of the algorithm. These criteria, in essence, establish a minimum density test which a data point must pass in order to be considered part of a cluster. In the case that the test is not passed, the point will be considered an outlier (cluster "-1"). An advantage of this method is that the methodology allows clusters to have a more abstract shape, while the previously cited methods have a bias towards proposing cycloid clusters;
- **OPTICS**: The OPTICS algorithm [20] is a DBSCAN variation that maintains an hierarchy of points, with variant neighborhood distance, thus addressing a DBSCAN weakness: cluster detection in group of points with varying density.

The tested clustering techniques generate a cluster scheme by studying around a 100 variables. In order for the user to properly view and understand what each cluster means, however, three variables need to be selected by default in order to built a three dimensional scatter plot. To achieve this two methods were tested, Absolute Correlation Sum (ACS) and ANOVA. The Absolute Correlation Sum, being proposed in this work, is given by:

$$ACS(v) = \sum_{c \in C} |r_{vc}| \tag{3}$$

Where $v \in V$ are the variables available to be used as axis in the plot, $c \in C$ are the variables that represent to which cluster a data point belongs, such that $c \in \{0, 1\}$ and $\sum_{c \in C} c \leq 1$. To generate the graph it suffices to take the three variables (two for a bidimensional plot) with the highest ACS.

The alternative, Analysis of Variance, commonly referred to as ANOVA [21], is a technique used by the statistics community to test relevant hypothesis. The main premise is that it is possible to explain the variance of a dependent variable through the variance of the independent variables. It is possible to use it to study the relationship between process variables and belonging to specific clusters.

III. Case Study

Given FPDA's scope, the assessment of how appropriately a set of clusters explains a dataset can be difficult to quantify, particularly due to the underlying needs of the user that requests the clustering, which can vary from scenario to scenario. Nevertheless two metrics are used to assess the quality of a clustering scheme, being presented below.

Silhouette Score [22] is a metric that measures how well defined and split a set of clusters are. It is defined by:

$$S = \frac{1}{|I|} \sum_{i \in I} \frac{b_i - a_i}{\max(a_i, b_i)} \tag{4}$$

Where S is the average Silhouette Score, I is the set of samples (data points), i is a data point such that $i \in I$, a_i is the average intra-cluster distance of i and b_i is the average inter-cluster distance to the nearest cluster.

The Weighted Silhouette Score is proposed, given by:

$$W = \frac{p_c (S + 1)}{2(1 + wn_c)} \tag{5}$$

The proposed metric makes the following alterations to the behaviour of the Silhouette Score: the interval of the result is altered from $[-1, 1]$ to $[0, 1]$, there is a new parameter n_c, which refers to the number of clusters used, and alongside parameter w, allows the user to penalize clustering schemes with large amounts of clusters; The p_c parameter refers to the amount of not clustered data (represented as belonging to the "-1" cluster), penalizing clusters that leave too many data points out. A value of w such that $w = 0$ makes it so that the penalty to large amount of clusters is deactivated.

IV. Results

The results observed were found satisfactory. Fig. 3, 4 and 5 showcase some of the clustering results (some details were redacted due to sensitive information).

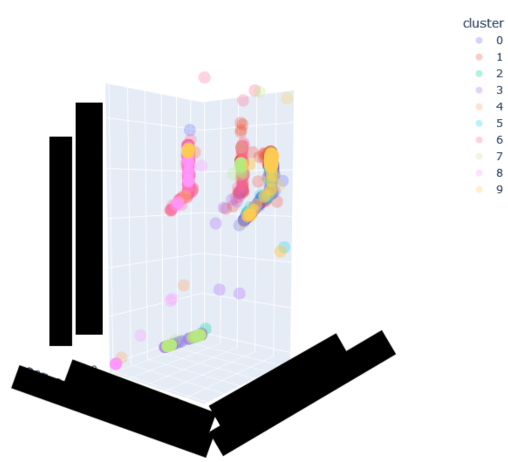

Fig. 3. P1 clustering example, done with the K-Means algorithm. 10 clusters were found. The process variables selected to compose the scatter plot were selected by the ANOVA method, those being Injection Compressor Discharge Pressure, Injection Compressor Suction Pressure and Main Compressor Discharge Temperature. Some details were redacted due to sensitive information.

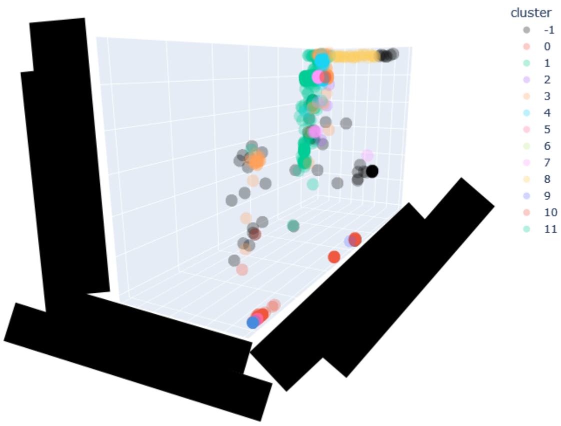

Fig. 4. P2 clustering example, done with the Mean-Shift algorithm. 12 clusters were found, with outliers being attributed the cluster ”-1”. The process variables selected to compose the scatter plot were selected by the ANOVA method, those being Flow rate in the Deaerator’s entrypoint, Active Power in the Sea Water Injection Booster Pump and Water flow rate in the discharge line of the Booster Pump. Some details were redacted due to sensitive information.

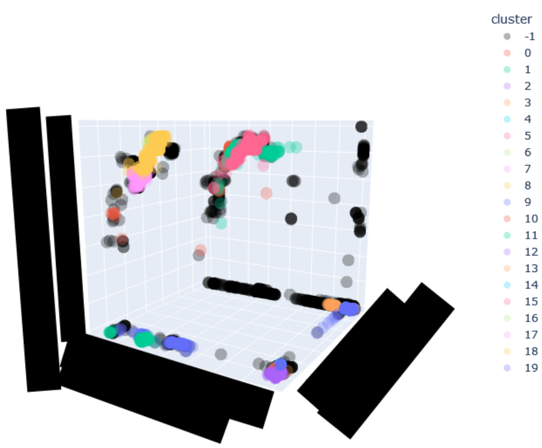

Fig. 5. P3 clustering example, done with the OPTICS algorithm. 20 clusters were found, with outliers being attributed the cluster ”-1”. The process variables selected to compose the scatter plot were selected by the ACS method, those being Active power of the Sea Water injection booster pump, Suction line of the main water injection pump and Exit flow rate of the export Compressor. Some details were redacted due to sensitive information.

Tables IX, X, XI present the results for the three platforms, with input parameters such that $w = 0$. The K-Means technique is the only one that requires the number of clusters to be used as an input parameter; in its case, the number of clusters presented is the number such that the Weighted Silhouette Score is maximized in the interval [3, 10]. All other algorithms automatically infer the number of clusters.

TABLE IX
P1

	K-Means	Mean-shift	DBSCAN	OPTICS
N. Clusters	10	1	91	22
Sil. Score	0.29	-1.00	0.75	0.68
W. Sil. Score	0.64	0.00	0.61	0.45
Clust. Percent	100.00	96.90	70.40	53.80

TABLE X
P2

	K-Means	Mean-shift	DBSCAN	OPTICS
N. Clusters	4	12	17	9
Sil. Score	0.57	0.56	0.39	0.47
W. Sil. Score	0.78	0.76	0.60	0.42
Clust. Percent	100.00	97.70	86.80	57.60

TABLE XI
P3

	K-Means	Mean-shift	DBSCAN	OPTICS
N. Clusters	10	2	90	20
Sil. Score	0.31	0.33	0.68	0.63
W. Sil. Score	0.65	0.65	0.56	0.38
Clust. Percent	100.00	97.80	66.70	46.80

It should be observed that while different clustering techniques might present unique output characteristics, sometimes desirable, sometimes not, the K-Means algorithm presented, for all cases, the highest Weighted Silhouette Score.

While in no means trying to be a full benchmark, table XII shows the processing time, in seconds, observed during the tests. A total of 24 runs were executed: eight for each platform’s history, from which two for each technique (one training for each visualization strategy). It can be observed that the time to select the initial variables for visualization is negligible in relation to the clustering time. Note that the ”Total” column indicates not only the clustering and variable selection routines, but the entire process, including, for instance, generating the interactive graphs seen in Fig. 3, 4 and 5 and miscellaneous data manipulation.

V. Conclusion

This paper presented the algorithms employed in the equipment modelling and clustering modules of FPSO Power Demand Analytics (FPDA), a Python-based solution for supporting FPSO electrical system modelling, focusing on the

TABLE XII
AVERAGE PROCESSING TIMES [SECONDS]

	Clustering	Select Var	Total
K-Means	12.57	0.15	13.25
Mean-shift	57.99	0.17	58.28
DBSCAN	1.02	0.32	1.60
OPTICS	67.47	0.19	67.80

application of the clustering module. Both modules (equipment modelling and clustering) were found to be adequate and sufficiently accurate.

In regards to the equipment modelling techniques, accurate projections were observed for most models, with absolute percentage errors rarely exceeding the 5% mark.

In regards to the clustering module, the K-Means algorithm presented the best WSS scores for all platforms, being thus recommended as the standard algorithm for this application. It should be noted, however, that the quality of a clustering scheme is subjective to the user's needs. This justifies the recommendation for iterative use by the users.

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