Fuzzy Multi-Criteria Decision Making Methods with Uncertainty Scenarios

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Abstract. Fuzzy Multi-criteria decision making methods have been provided to help the decision-makers in their complex decisions about future uncertainties. Taking into consideration uncertainties such as vagueness and future scenarios, this paper aims to apply the methods Fuzzy-MultiMoora, Fuzzy-Topsis Linear, Fuzzy-Topsis Vector, Fuzzy-Vikor and Fuzzy-Waspas in a Hydrothermal Dispatch problem. Five scenarios were evaluated by varying hydrology and energy demand parameters, from very pessimistic to very optimistic. Two decision makers made explicit their preferences weighting three criteria: Cost, Rationing and Distance. The normalized fuzzy numbers were calculated using the concept of alpha-cuts. Finally, the indexes were aggregated into a final ordering considering weights for the methods based on the Kendall tau distance. The best solutions were compared in relation to the criteria. It was observed that these solutions presented good results in all scenarios evaluated.

Keywords: Fuzzy MCDM; Uncertainty Scenarios; Hydrothermal Dispatch.

1 Introduction

Multi-criteria decision making (MCDM) methods have received much attention in last decades [16, 17]. It has grown as a part of operations research and provides mathematical and computational support for specialists who deal with complex decision problems. Although they are very useful, the multi-criteria problems generally deal with uncertainties in a number of ways and these methods needed to be adapted to deal with vagueness and human imprecision. To overcome this problem, Fuzzy sets theory (FST) [25] provided an original way to deal with this problem by adoption of subjective preferences instead of only numerical values (crisp numbers).

As pointed out in [14], after the optimisation process, commonly exist a number of efficient solutions which are considered as candidates of final decision-making problem. Then, decision-makers need to decide which alternatives are the best, considering a set of preferable criteria, which are usually conflicting with each other. In order to help the decision-maker consider incomplete or non-obtainable information, FST combined with MCDM methods is revealed as a good way to solve this problem. The flexibility of fuzzy multi-criteria decision making (FMCDM) methods have been reflected in the literature considering the exponential growth of published papers. Two important researches provided a state-of-the-art review of the applications and methodologies. In [16], the authors focus on classical MCDM methods from 2000 to 2014. Later, in [17], they investigated the FMCDM techniques from 1994 to 2014. This clearly shows that methods, either classical or fuzzy, can be applied in a wide variety of application fields.

It is possible to note that many researches have focused on combining methods to solve real-life problems. It is known that each method can rank the alternatives in different ways, based on decision-maker's preferences. Thus, one distinct way to capture the best of each method is to aggregate these different rankings and classify them in a final ordering. Moreover, it has been suggested the evaluation of multi-criteria problems considering different scenarios [9]. Previous works have shown that the combination of different MCDM methods can improve the results when compared with the application of a single one [4, 13, 18]. Also, uncertainties have been incorporated into the assessments to further improve decision making, as presented in [8, 9].

In this paper, the authors propose the use of different methods, namely Fuzzy-MultiMoora [2, 3], Fuzzy-Topsis Linear [7, 23], Fuzzy-Topsis Vector [10, 21], Fuzzy-VIKOR [20] and Fuzzy-Waspas [22], in a practical Hydrothermal Dispatch [19] problem considering maintenance outages. A set of 80 initial solutions were obtained, which represent maintenance plans of the generation units. From these solutions, five scenarios, from very pessimistic to very optimistic, were evaluated by varying hydrology and energy demand parameters. A conservative decision was taken into consideration in this case. Then the Kendall tau rank distance [15] was used to measure the rankings. Finally, a final ordering was carried out and some alternatives were compared to evaluate the final decision.

The rest of the paper, therefore, proceeds as follows: Section 2 shows some basic concepts of fuzzy sets and provided a brief review about FMCDM techniques, focusing on aforementioned methods. Additionally, it summarizes some types of uncertainties in decision environment. Section 3 explains the methodology adopted and explain succinctly the power system problem used. Section 4 presents the results and a discussion about the final ranking. Section 5 provides the conclusion and future works.

2 Contextualization

2.1 Preliminary concepts of fuzzy sets

FST is an efficient tool for modeling imprecision and vagueness and has had good results in several applications, see [17]. FST is used to capture imprecise and subjective information and transform them into a numerical format, aiming at the approximate reasoning.

Preliminary concepts of fuzzy sets are necessary to understand the rest of the paper (adapted from [14]):

1. A fuzzy set is defined as $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) | x \in U\}$, being U the Universe of Discourse (UoD), x is an element in U, $\tilde{A}$ is a fuzzy set in U, $\mu_{\tilde{A}}(x)$ is the membership function (MF) $\mu_{\tilde{A}}$ at x.
2. A Triangular Fuzzy Number (TFN) can be denoted as $\tilde{A} = (l, m, u)$, where m is the central value ($\mu_{\tilde{A}}(m) = 1$), l is the left spread and u is the right spread.
3. A TFN is a real fuzzy number $\tilde{A}$ which possesses the following properties:
   (a) $\mu_{\tilde{A}} = 0$, $\forall x \in [-\infty, l]$
   (b) $\mu_{\tilde{A}}$ is strictly increasing on $[l, m]$
   (c) $\mu_{\tilde{A}}$ is strictly decreasing on $[m, u]$
   (d) $\mu_{\tilde{A}} = 0$, $\forall x \in [u, \infty]$
   where $l \leq m \leq u$.
4. The α-cut of $\tilde{A}$ is a crisp subset of X and it is denoted as $[\tilde{A}]_\alpha = \{x | \mu_{\tilde{A}} \geq \alpha\}$ where $\mu_{\tilde{A}}(x)$ is the MF of $\tilde{A}$ and $\alpha \in [0, 1]$.
5. Let $\tilde{A}$ be a TFN. $\tilde{A}_\alpha$ is a non-empty bounded closed interval and can be expressed by $\tilde{A}_\alpha = [[\tilde{A}]^L_\alpha, [\tilde{A}]^U_\alpha]$, where $[\tilde{A}]^L_\alpha$ and $[\tilde{A}]^U_\alpha$ are its lower and upper bounds, respectively.
6. If $\tilde{A} = [[\tilde{A}]^L_\alpha, [\tilde{A}]^U_\alpha]$, then by choosing $\alpha = 1$ we can identify the central value of $\tilde{A}$, and by $\alpha > 0$ the left and right spreads of $\tilde{A}$.
7. If $\tilde{A}$ is a TFN and $[\tilde{A}]^L_\alpha > 0$ and $[\tilde{A}]^U_\alpha \leq 1$ for $\alpha \in [0, 1]$, then $\tilde{A}$ is called a normalized positive triangular fuzzy number.
8. Linguistic value is a variable whose values are expressed as subjective terms. For example, "importance" is a linguistic variable whose values include EL (Extremely Low), VL (very Low), L (Low), H (High), VH (Very High), EH (Extremely High). Considering these linguistic values represented as TFN they can expressed as $EL = (0.0, 0.0, 0.2)$, $VL = (0.0, 0.2, 0.4)$, $L = (0.2, 0.4, 0.6)$, $H = (0.4, 0.6, 0.8)$, $VH = (0.6, 0.8, 1.0)$, $EH = (0.8, 1.0, 1.0)$.

2.2 Fuzzy Multi-criteria Decision Making Methods

The literature contains a great number of classifications of FMCDM tools. For instance, in [17] the authors indicated 403 papers addressed to several FMCDM. It is important to highlight that hybrid FMCDM in the integrated method was ranked as the first tool among other tools and approaches.

Gul et al. [11] conducted a literature review about VIKOR (*VlseKriterijumska Optimizacija I Kompromisno Resenje*, means multi-criteria optimization and compromise solution) method and its fuzzy extensions. They summarized 343 papers classifying them into 13 categories. Fuzzy-Vikor had great relevance, especially in design, mechanical, engineering and manufacturing category. Additionally, another research, [18], provided a systematic review of this method. Integrated techniques with Vikor and fuzzy-Vikor were highlighted as being hardly employed.

Behzadian et al. [4] provided a review of Technique for the Order Preference by Similarity to Ideal Solution (TOPSIS) applications and 266 papers were summarized. Fuzzy set approach and group decision-making approach were the top two on the distribution of techniques. Many studies have proposed extensions of Topsis method, originally provided in [23]. They differ in the type of normalization for the method. Thus, consider that Topsis-Linear follows the original proposal in [23] and Topsis-Vector is an extension of this method, but considering vector normalization. These differences were explained and can be easily observed in [21]. For simplicity, we consider in this paper these two approaches as two different methods.

Constantly new methods have been created as alternatives to multi-criteria problems. Among them we can mention Weighted Aggregated Sum–Product Assessment (WASPAS) method [26] and Multi-objective optimization by ratio analysis (MOORA) [5] plus the full multiplicative form (MULTIMOORA) [6]. The former is a value measurement model [3], which provides complete aggregation of the weighted sum and weighted product approaches. The latter, as well as Topsis and Vikor methods, is a method based on reference point technique [3].

As a note, for simplicity, we just mention the origin and reference papers of the methods evaluated in this research and some relevant research that addresses them. Their stepwise descriptions can be easily found in references and literature on MCDM methods in general.

2.3 Treating Uncertainties in Decision Problems

The real-world problems generally deal with uncertainties in a great number of ways [17]. Hence, the consequences of an action are often unknown because they depend on future events [8], then facilitating decisions under conditions of uncertainty requires a choice about how this uncertainty is to be modeled [9].

Several formats exist for representing uncertainty, and for each of these formats, many prescriptive decision models have been developed [9]. Among the five types of uncertainties reported in Durbach and Stewart review [9], fuzzy theory and multiple scenarios were highlighted. FST is often used to treat mainly vagueness and imprecision on decision-maker's preferences. Scenarios deal with incomplete descriptions of how the future might unfold. It considers the question of scenario construction and desirable features of scenarios [9].

Moreover, the authors also emphasized the importance of comparing and aggregating results from different scenarios. Namely, this aggregation process takes into account the use of relative weights in the performance of the scenario. Models which evaluate scenario analysis are one of the three simplifications that Durbach and Stewart [8] provided regarding the ways of handling uncertainties in the multi-criteria analysis. In this proposal, the evaluation of the alternatives A_i is given by Equation (1)

$$U_i^{(scen)} = \sum_{k=1}^{N_s} \left[w_{s_k} \sum_{j=1}^{J} v_j u_j (z_{ij}^{(s_k)}) \right] \tag{1}$$

where U_i is the expected utility of alternative A_i, v_j is an attribute importance weight, s_k refers to a specific scenario, $z_{ij}^{(s_k)}$ is the evaluation of alternative A_i

on criteria C_j in scenario s_k, w_{s_k} is the weight associated with scenario s_k and N_s is the number of scenarios [8].

3 Methodology Proposed

3.1 Case Study

As suggested in previous works [1], [8] and [9], we take into consideration the evaluation of different scenarios based on the variation of parameters in a practical power energy problem modeled in [19]. The authors investigated a classical Hydrothermal Dispatch (HTD) problem considering maintenance outages. They propose twelve heuristics based on different criteria, such as load demand, thermal plants cost, water inflows, distance from base plan etc. As a numerical example, the authors used a system composed of three hydro plants and 2 thermal plants divided into 22 machines. The time horizon was considered 52 weeks (1 year), it means, short-term, and four types of maintenance were evaluated.

The monitoring task of power generation plants consists of a decision-making tool [24, 1]. The result, as several optimization problems and reported in [14], was a number of efficient solutions which we considered the candidates of final decision making problem. In the problem reported by Martínez et al. [19], the authors found 80 feasible solutions, i.e. 80 different maintenance plans of the generation units. We use the same sample of 80 solutions, but we stop the algorithm before its convergence to obtain solutions more diversified in terms of cost and rationing. The procedure is described as follows.

3.2 Procedure

(a) *Step A: Scenarios*: The 80 solutions obtained directly from optimisation process were considered as the first scenario. The same set of solutions were re-evaluated subject to variations of hydrology and energy demand parameters. They differ by around 10% or 20% in these parameters. At the end, the following scenarios were obtained: S_1: Standard (from the optimisation process), S_2: Pessimistic, S_3: Very Pessimistic, S_4: Optimistic, S_5: Very Optimistic.

(b) *Step B: Fuzzy Methods*: As mentioned above, the fuzzy-MCDM methods considered were: M_1: Fuzzy-MultiMoora, M_2: Fuzzy-Topsis Linear, M_3: Fuzzy-Topsis Vector, M_4: Fuzzy-Vikor, M_5: Fuzzy-Waspas.

(c) *Step C: Criteria*: Three criteria were taken into consideration. C_1: operational cost, from the objective function of the optimisation problem. C_2: Rationing, a penalty for not supplying the energy demand, and C_3: Distance from the base maintenance plan. This criterion had the same values for all the scenarios.

(d) *Step D: Fuzzy Decision Matrix*: It is known that all the MCDM methods can be improved by decomposing the overall evaluation of alternatives into evaluations on a number of usually conflicting criteria relevant to the problem [9, 14].

(i) *Rating of alternatives*: To deal with the imprecision, the values from the three criteria were normalized to an interval [0, 1]. Then, triangular fuzzy sets equally spaced in that interval were defined to rating these alternatives. Table 1 summarizes these data. For fuzzification process, given the crisp value coming from the optimization problem, the normalized TFN were transformed using the concept α-cuts, i.e., $\tilde{N}_{ij} = (n_{ij}, a_{ij}, b_{ij})$, such that n_{ij} is obtained when $\alpha = 1$ and left and right spreads are $a_{ij} = n_{ij} - [\tilde{n}_{ij}]^L_{\alpha=0}$ and $b_{ij} = [\tilde{n}_{ij}]^U_{\alpha=0} - n_{ij}$, respectively.

Table 1. Linguistic variables for the ratings of the alternatives

Abbr.	Linguistic values	Fuzzy Number
EP	Extremely Poor	(0.0, 0.0, 0.1)
VP	Very Poor	(0.0, 0.1, 0.2)
PVP	Poor to Very Poor	(0.1, 0.2, 0.3)
P	Poor	(0.2, 0.3, 0.4)
PF	Poor to Fair	(0.3, 0.4, 0.5)
F	Fair	(0.4, 0.5, 0.6)
FG	Fair to Good	(0.5, 0.6, 0.7)
G	Good	(0.6, 0.7, 0.8)
GVG	Good to Very Good	(0.7, 0.8, 0.9)
VG	Very Good	(0.8, 0.9, 1.0)
EG	Extremely Good	(0.9, 1.0, 1.0)

(ii) *Weights*: Classically, the importance weight of the criteria was evaluated by using linguistic variables - see Table 2. At the end, these evaluations were aggregated and a single fuzzy ranking was used.

(iii) *Decision-makers*: Two decision-makers were consulted for this analysis. They evaluated the set of criteria using the linguistic values and these judgments were aggregated.

Table 2. Linguistic variables for the importance weight of the criteria

Abbr.	Linguistic values	Fuzzy Numbers
EL	Extremely Low	(0.0, 0.0, 0.2)
VL	Very Low	(0.0, 0.2, 0.4)
L	Low	(0.2, 0.4, 0.6)
H	High	(0.4, 0.6, 0.8)
VH	Very High	(0.6, 0.8, 1.0)
EH	Extremely High	(0.8, 1.0, 1.0)

(e) *Step E: Getting the rankings*: All the methods for all the scenarios were evaluated. At the end, 25 rankings were obtained. They represent the decision-maker's preference in respect to the evaluated criteria as well as evaluate uncertainty scenarios considering positive and negative forecasts for hydrology and energy demand.

(f) *Step F: Aggregating the scenarios*: In this step, we follow the models using scenarios provided in [8], see Equation (1). As we used different FMCDM methods we apply weights to the methods as well.

   (i) *Evaluation of alternatives*: In this case, it was considered the index of the alternatives in the rankings obtained by the FMCDM method.

   (ii) *Weight associated with fuzzy-method*: Predictably each method classified the 80 alternatives in different orders. To measure this, the Kendall tau rank distance [15] was invoked. This metric is useful because it counts the number of pairwise disagreements between two ranking lists. It is given by the Equation (2). The normalized distance is a value in the interval [0, 1].

$$K(\tau_1, \tau_2) = |(i,j) : i < j, (\tau_1(i) < \tau_1(j) \wedge \tau_2(i) > \tau_2(j)) \vee (\tau_1(i) > \tau_1(j) \wedge \tau_2(i) < \tau_2(j))| \quad (2)$$

   where $\tau_1(i)$ and $\tau_2(i)$ are the rankings of the element i in τ_1 and τ_2, respectively. $K(\tau_1, \tau_2)$ is equal 0 if the two lists are identical and equal 1 if one list is the reverse of the other.

   (iii) *Weight associated with scenarios*: Arbitrary weights were adopted for the scenarios, adopting a conservative attitude favoring, mainly, the nominal scenario. $w_{s_k} = 0.6, 0.2, 0.1, 0.2, 0.1$, for scenarios from 1 to 5, respectively.

(g) *Step G: Getting final ordering*: All the methods for each scenario were aggregated by multiplying the weight of the FMCDM method by their indexes. At the end, these values were added again, considering the five scenarios. Finally, the alternatives were reordered and the final ordering was provided.

4 Results and Discussion

Five scenarios were considered in this paper. They represent consistent narratives of how the future might unfold according to the parameters considered, i.e. hydrology and energy demand. For optimistic scenarios, high hydrology and low energy demand were considered, while in pessimist scenarios low hydrology and elevated energy demand were considered. For the standard scenario, the same set of 80 solutions obtained from the optimisation process was used.

For each one of these five scenarios $S_1...S_5$, a different FMCDM $M_1...M_5$ was applied, taking into consideration the three criteria $C_1...C_3$. Using the linguistic rating variable to assess the importance of the criteria, two decision-makers DM_1 and DM_2 explicit their preferences, according to Table 3.

To calculate the rating of the alternatives to construct the fuzzy decision matrix, the concept of α-cuts, i.e., $\tilde{N}_{ij} = (n_{ij}, a_{ij}, b_{ij})$ was used. At the end, the algorithms was executed and 25 rankings were obtained. For simplicity, the indexes of these 15 first alternatives were summarized in Table 4.

Table 3. Weights of the Criteria

	Linguistic rating		Fuzzy value		Aggregated
C_j	DM_1	DM_2	DM_1	DM_2	DM_{aggreg}
C_1	L	L	(0.2, 0.4, 0.6)	(0.2, 0.4, 0.6)	(0.2, 0.4, 0.6)
C_2	L	H	(0.2, 0.4, 0.6)	(0.4, 0.6, 0.8)	(0.2, 0.5, 0.8)
C_3	EL	VL	(0.0, 0.0, 0.2)	(0.0, 0.2, 0.4)	(0.0, 0.1, 0.4)

Source: Authors, 2017

Table 4. Indexes of alternatives for each FMCDM in each scenario

	M_1					M_2					M_3					M_4					M_5				
A_n	S_1	S_2	S_3	S_4	S_5	S_1	S_2	S_3	S_4	S_5	S_1	S_2	S_3	S_4	S_5	S_1	S_2	S_3	S_4	S_5	S_1	S_2	S_3	S_4	S_5
A_1	51	51	51	24	50	45	56	53	13	48	45	42	41	12	42	43	52	54	7	47	51	51	52	17	50
A_2	12	16	22	2	20	16	24	20	1	14	8	18	15	2	20	16	25	20	1	13	13	19	28	3	20
A_3	29	15	14	1	4	51	17	51	2	20	21	7	4	1	2	48	10	42	2	16	37	12	5	1	5
A_4	9	8	28	10	8	18	11	28	6	6	5	9	28	8	7	17	16	29	6	6	11	11	34	14	8
A_5	24	14	61	34	46	8	33	31	18	47	31	25	64	41	50	7	38	40	19	50	21	21	60	37	47
A_6	2	18	1	3	1	6	2	6	3	1	2	19	1	3	1	5	2	6	4	1	5	14	3	6	1
A_7	20	3	16	68	22	31	13	1	68	22	14	4	65	68	22	30	8	1	73	21	22	1	1	71	23
A_8	13	6	67	49	13	24	10	66	41	12	11	8	67	42	15	24	11	66	31	10	15	8	67	27	13
A_9	10	11	78	46	9	49	14	78	32	7	6	14	78	48	10	18	17	77	21	7	12	13	77	43	9
A_{10}	21	9	73	53	6	38	6	73	44	13	16	5	72	55	5	37	5	74	40	14	28	10	73	57	6
A_{11}	1	1	65	4	5	3	1	47	4	2	1	1	68	4	6	3	1	52	3	2	2	4	65	5	4
A_{12}	6	2	79	41	3	15	3	79	25	4	4	3	79	44	3	15	3	79	22	4	8	5	79	42	3
A_{13}	3	26	33	13	31	1	39	7	30	38	28	47	59	36	47	1	39	7	39	38	1	31	22	13	29
A_{14}	25	24	5	16	54	5	31	3	21	62	52	48	25	37	60	6	35	3	28	63	6	26	17	19	53
A_{15}	63	32	3	15	64	63	66	2	17	74	67	51	18	34	66	64	63	2	15	73	58	35	16	15	63

Source: Authors, 2017

To aggregate the scenarios in order to get a final ordering, we used the index that each alternative obtained in FMCDM instead of using their properly ranking. This is shown in Table 4. Considering the five different FMCDM, the Kendall tau rank distance [15] was adopted because this metric is used to indicate the degree of similarity between different lists. The lower the value of the metric, the closer the methods classified the alternatives. Table 5 shows these measures.

However, it can be noted that this measure indicates the degree of similarity between τ_1 and τ_2, which means the indexes provided by the FMCDM. The tau distance of each method was compared to the sum of the distances of the other methods. Therefore, to indicate the weight of each method, we use the inverse of this measure. Thus, the smaller the tau distance given by the metric, the greater the weight of the method in the aggregating of results.

For the scenarios, arbitrary weights were adopted favoring the standard scenario S_1. This is because, being a practical problem, the data was obtained with the generating company (UPME - Mining and Energy Planning Unit of Colombia, in English). So it was believed that it was the most likely scenario.

Table 5. Kendall tau distance among the FMCDM

Fuzzy-Methods		Scenarios				
M_1	M_2	S_1	S_2	S_3	S_4	S_5
Fuzzy-MMoora	*Fuzzy-Topsis Linear*	0.1076	0.2013	0.1127	0.1636	0.0348
	Fuzzy-Topsis Vector	0.0684	0.0823	0.1415	0.0962	0.0418
	Fuzzy-Vikor	0.1051	0.1759	0.0861	0.2595	0.0427
	Fuzzy-Waspas	0.0699	0.0532	0.0696	0.1095	0.0291
Fuzzy-Topsis Linear	*Fuzzy-Topsis Vector*	0.1570	0.2203	0.2541	0.1642	0.0532
	Fuzzy-Vikor	0.0127	0.0570	0.0285	0.0997	0.0180
	Fuzzy-Waspas	0.0927	0.2259	0.1348	0.1649	0.0519
Fuzzy-Topsis Vector	*Fuzzy-Vikor*	0.1481	0.1810	0.2275	0.2468	0.0446
	Fuzzy-Waspas	0.0668	0.1158	0.1801	0.1304	0.1304
Fuzzy-Vikor	*Fuzzy-Waspas*	0.0851	0.2070	0.1177	0.2222	0.0604

Source: Authors, 2017

Finally, the final ordering was calculated. First, the weights of each FMCDM was multiplied by their respective indexes. Then, these values were added again. Finally, the alternatives were reordered and the final ranking was obtained. This final ranking is shortened in Table 6.

Table 6. Final ranking aggregated

Ranking	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	...	80
A_n	6	11	4	2	13	12	19	80	9	69	8	64	14	3	18	29	78	23	5	61	...	37

Source: Authors, 2017

4.1 Discussion

Usually, the papers addressed to hydrothermal dispatch aim at the minimization of an objective function, given the constraints. In the case investigated in this article, the problem considered maintenance outages. If only the cost criterion is taken into account, the option with the lowest value obtained is chosen and implemented. However, these problems take into account others criteria. Therefore we have also defined the Rationing and Distance criteria of the base plan.

Observing Table 6 and analyzing the first three alternatives obtained with the aggregation of the rankings, it is shown that these presented good indexes in most evaluated scenarios. The A_6 alternative was classified in the top 10 in 22 scenarios, A_{11} in 20 scenarios, A_4 in 12 and A_2 and A_{13} in 6 scenarios. The alternatives A_6, A_4, A_2 and A_{13} were still stable in the ranking positions for all scenarios, while A_{11} was poorly evaluated specially in scenario S_3. Table 7 summarizes the real values of these alternatives from the optimisation process in each scenario. Just consider that the values of criterion C_3 were the same for all scenarios.

Table 7. Crisp values of the best alternatives classified in the final ordering

S_k	C_j	A_6	A_{11}	A_4	A_2	A_{13}
S_1	C_1	1.208.956,8	1.210.706,0	1.214.833,1	1.218.879,6	1.230.436,1
	C_2	0,0	0,0	0,0	0,0	0,0
S_2	C_1	1.410.806,6	1.458.036,4	1.464.998,0	1.470.220,7	1.478.693,1
	C_2	81.328,1	81.327,9	81.328,0	81.328,1	81.328,1
S_3	C_1	7.107.905,0	7.158.482,0	7.122.344,0	7.127.141,0	7.136.581,5
	C_2	22.044,9	22.044,9	22.045,0	22.045,0	22.045,0
S_4	C_1	1.325.575,6	1.362.728,5	1.354.278,9	1.346.907,3	1.342.081,2
	C_2	19.021,7	18.316,8	19.061,8	18.327,9	20.100,9
S_5	C_1	855.137,7	855.137,7	858.671,1	860.956,7	86.6613,4
	C_2	0,0	0,0	0,1	0,0	0,0
S_{all}	C_3	233	220	314	311	99

Source: Authors, 2017

For this evaluation, it was considered a more conservative attitude, favoring scenario S_1. And previously the alternatives with high costs either in the objective function or of rationing were more penalized, according to the judgments regarding the criteria. In addition, other analyses can be easily performed by varying scenarios or weights for the methods, as shown in Table 4. The authors believe that these results may encourage further research dealing with uncertainties, hydrothermal dispatch and decision theory.

5 Conclusions

This research dealt especially with two types of uncertainties. Vagueness as to the judgment of alternatives and criteria and uncertainty scenarios. In addition, five FMCDM were considered in this analysis, which required a way to evaluate the ranking obtained by each of them. For the alternatives and criteria, it was provided by using the concepts of α-cuts in order to obtain the normalized fuzzy numbers. Two decision-makers evaluated the criteria. Classically, they used linguistic rating variable to assess the importance of the criteria. In addition to the standard scenario arising from the optimization problem, four other scenarios were considered by varying hydrology and energy demand parameters. Thus, it was possible to include in the analysis the uncertainty as to the variation of these parameters in both the best and the worst case. The aggregation of indexes allowed to absorb all these preferences and risks and generate a final ordering.

Scenarios, planning and building, are especially powerful for exploring the futures [12]. It provides information on losses and gains considering the probability of occurrence of that scenario. More conservative or aggressive judgments can be provided either according to the knowledge about the problem or based on decision-maker's preference. It is known that the good MCDM method is one that reflects the decision-maker's preference, thus it must reflect these preferences in final ordering. When different scenarios are combined, those preferences are also reflected in the modeling process and may help balance the assessment.

It should be noted that the conclusions provided in this paper do not represent absolute certainty in asserting that the solutions are definitive. However, we clearly believe that it reflects the decision-maker's preference throughout the post-optimization process. However, it can undergo changes depending on the evaluations of the decision-makers in their judgments.

Hybrid techniques and type-2 fuzzy approaches are becoming increasingly important. They are based on previously methods, such as used here and their modifications. New methods have also emerged in the literature, which allows new analyzes and comparisons. In addition, we use a classical hydrothermal dispatch problem, but the approach applied in this paper may be extended to any other problem in operational research which deals with crisp outcomes and can have the parameters of the optimization model modified.

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