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A Generalization of Graded Response Formal Neurons

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Abstract

We introduce a simple generalization of graded response formal neurons which presents very complex behaviour. Phase diagrams in full parameter space are given, showing regions with fixed point, periodic, quasiperiodic and chaotic behaviour. The diagrams also represent the possible time series implementable by the simplest feed-forward network, a two-input single-layer perceptron.

1 Introduction

Although convergent dynamics (relaxation to fixed points) has been a dominant theme in artificial neural networks studies [1], oscillatory behaviour in network models is receiving growing atention. Oscillatory *synchronized* dynamics has been observed at different biological functional levels, for example in the visual cortex [2] and sensoriomotor cortex [3]. Such behaviour has attracted great attention since it seens to be an important cue for solving the feature-binding problem (how the brain links different perceptual signals to the same object).

This has prompted some researchers, which have previously worked with networks of formal neurons (Hopfield attractor networks with MacCulloch-Pitts or graded response elements), to consider with renewed interest models of coupled oscillators [4, 5, 6], since it has not been clear how to extend the previous Hopfield paradigm to collective synchronization phenomena.

Other researchers, from the dynamical systems community, have proposed coupled maps lattices (CML) as an alternative paradigm to consider synchronization, chaos and other spatio-temporal phenomena in biological neural networks [7]. The most studied models use as basic element the logistic map, perhaps because this map is simple and well known.

In this work we propose a new formal neuron which is a n-dimensional map (then, networks of these elements lie in the coupled maps paradigm). However, this map is a very natural extension of the graded response neurons popularized by Hopfield [8]. We call this map a *dynamical perceptron* (DP). A network of totally connected DPs constitutes a globally coupled maps (GCM) system [7] which is a simple generalization of Hopfield networks of graded response neurons [9, 10].

This is a very *preliminar* study where we explore some aspects of the model. In section 2 we introduce the dynamical perceptron map and in section 3 we present phase diagrams in parameter space for the $n = 2$ case. Section 4 contains our conclusions and outlook.

2 The model

Neuron models have two deal with to competing constraints: biological realism and mathematical tractability. Some models (for example, Hodgkin-Huxley neuron) stress the first one and others (formal neurons) achieve the second one only throught dramatic simplifications. Hodgkin-Huxley neurons present a varied repertoire of dynamical behaviours depending on its parameter values, but are not easy to study analytically or even computationally. On the other hand formal neurons most considered in the literature are structurally simple but have no intrinsic dynamics.

We adopt the physicist approach where, for studing the *necessary* (in contrast with the *sufficient*) requisites for the appearance of some collective behaviour in networks, the simplest elements with the simplest interactions should be first considered. But we want to propose a formal neuron which, althought structurally simple, presents complex dynamical behaviour at least *qualitatively* similar to real neurons. Unlike other proposals, however, our model stays within the formal neuron paradigm popularized by Hopfield, since it is a simple generalization of the graded response neuron widely used in the literature.

The two most considered formal neurons are the *McCulloch-Pitts neuron* (a binary variable like an Ising spin) and the *graded response neuron* [8]. The state variable of the graded response neuron is devised to describe not the single action potentials (which are all-or-none events) but the average frequency of these spikes. This average frequency we will call the *activity* $V(t)$ of the neuron. The neuron activity response to external inputs can be approximated by a sigmoidal curve. This is one of the reasons behind the popular choice of the hiperbolic tangent as the formal neuron transfer function [1].

Consider a time series of this average firing activity (how this time series can be obtained is not our concern now). Our task will be to find a simple model which describes such time series. A simple and general approach to this modelling problem, which we will follow here, is to use a feed-forward artificial neural network (FANN) to emulate the system behaviour [1].

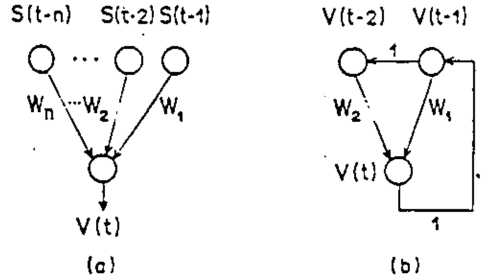

Figure 1: a) single-layer perceptron; b) dynamical perceptron with $n = 2$.

It is well known that FANNs with one layer of hidden units can approximate any continuous function. Then, we can in principle to train a feed-forward net to reproduce the biological neuron time series (a somewhat self-refferential application of neural nets!). Once we find such FANN (which will be a formal neuron model) we can couple various of them to form a large net of such formal neurons. At the end of this process we will achieve a large, complicated and theoretically intractable system.

Well, let's simplify our task. We ask for *the simplest feedforward net which qualitatively models the time series*. By *qualitatively* we mean a very weak requeriment: we want a formal neuron which presents, like biological neurons, not only graded response but also oscillatory behaviour in certain circumstances.

A single-layer perceptron (a feedforward net without hidden units) can do this and has the advantage of being a very well studied system (see figure 1a). The perceptron learns the time series $\{S(t)\}$ by using *examples*, that is, input-output pairs $(\mathbf{X}_t, Y_t)$. The desired output is the value of the time series at time t, $Y_t \equiv S(t)$. The input may be the previous n values of the time series $\mathbf{X}_t \equiv \{S(t-1), \ldots, S(t-n)\}$. Then, the examples are generated simply by slipping a window of lenght n over the data. The perceptron output will be given by [1]

$$\begin{aligned} V(t) &= g\left(\mathbf{W}.\mathbf{X}_t - \theta\right) \\ &= g\left(\sum_{\tau=1}^{n} W_\tau S(t-\tau) - \theta\right) \end{aligned} \quad (1)$$

where $\mathbf{W} = \{W_1, \ldots, W_n\}$ is the weight vector which defines the perceptron, the constant θ is the so called *bias* term and $g(x)$ is a non-linear continuous sigmoidal function.

After the trainning over a time series (remember that no perfect reprodution is required) we can create a dynamical model from the perceptron so obtained (we will call this system a *dynamical perceptron*, see figure 1b). To do this we simply send the perceptron output to the last input node and transfer the value present at each input node to its left neighbour (the extreme left value being discarded). That is, the dynamical perceptron output will be given by the following n-order recurrence equation

$$V(t) = g\left(\sum_{\tau=1}^{n} W_\tau V(t-\tau) - \theta\right) \qquad (2)$$

We will specialize the transfer functions $g(x)$ to the family of sigmoidal functions

$$g(x) = c_1 + c_2 \tanh(\gamma x) \qquad (3)$$

where c_1 and c_2 are constants. The usual sigmoid with image in the $[0,1]$ interval is obtained with $c_1 = c_2 = 1/2$. The hiperbolic tangent, of course, is given by $c_1 = 0$ and $c_2 = 1$. All these maps are topologicaly conjugated to the tanh one throught a change of variables $V' = (V - c_1)/c_2$ giving

$$V'(t) = \tanh\left[\gamma'\left(\sum_{\tau=1}^{n} W_\tau V'(t-\tau) - \theta'\right)\right] \qquad (4)$$

with the transformed variables

$$\gamma' = c_2\gamma, \qquad \theta' = \frac{\theta}{c_2} - \frac{c_1}{c_2}\sum_{\tau=1}^{n} W_\tau \qquad (5)$$

We will concentrate our attention in the *simplest* non-trivial dynamical perceptron, that is, the one with $n = 2$. We also note that a dynamical perceptron with $n = 1$ has been proposed as a formal neuron by Pasemann [11]. However, the $n = 1$ model has very poor behaviour, exhibiting only fixed points or two-cycle as attractors. Networks of such neurons have been considered by statistical physiscists [9].

The 2-D map, instead, presents very rich behaviour (fixed points, limit cycles, quasiperiodic and chaotic attractors, coexisting attractors etc.) which can be used to mimic some known behaviours of real neurons. It also allows the modelling of time-dependent neuron responses (habituation, sensitization) because the internal parameters $\mathbf{W}$, and not only the external synapses, can suffer a learning process.

We write the recurrence equation for the 2-dimensional DP as

$$V(t) = \tanh\left[\frac{V(t-1) - \kappa V(t-2) + H}{T}\right] \qquad (6)$$

where

$$T = \frac{1}{\gamma' W_1}, \quad \kappa = \frac{-W_2}{W_1}, \quad H = I - \theta', \qquad (7)$$

where we have allowed the possibility of an external input I and γ' and θ' are given by eq. (5). This is the simplest neural network model of a dynamical system and the full phase diagram in the variables (T, κ, H) gives all the possible time series implementable by this architecture.

The choice of these variables is motivated by convenience in the visualization of the phase diagram as well to connect the 2-D dynamical perceptron with previous literature. This map (for $H = 0$) has been studied in the contex of statistical mechanics models of magnetically modulated materials by Yokoi, Oliveira and Salinas [12], where $V(t)$ describes the magnetization on the t^{th} layer of an Ising model with competing interactions in a Bethe lattice. Recently, Yokoi and one of us (MHRT) have studied the map for constant and non-zero H [13].

3 Phase Diagrams

The first diagram (fig. 2, 3) refers to the dynamical behaviours in the plane T vs κ for $g(x) = tanh(x)$. The main periodic phases are shown and their labels $q/2\pi = P/Q$ denote their periods Q (the number of points after which the series repeats) and the number of crests P within a period (q is the 'modulated phase wave number'). Quasiperiodic phases have $q/2\pi$ irrational. In the region 0 there are only trivial fixed points. In the region 1 two fixed points coexist for all values of T and κ, and the neuron states are the solutions of $V = \tanh((1-\kappa)V/T)$. The convergence to each fixed point depends on the initial conditions $V(t=0)$ and $V(t=1)$. Phase 1/2 is

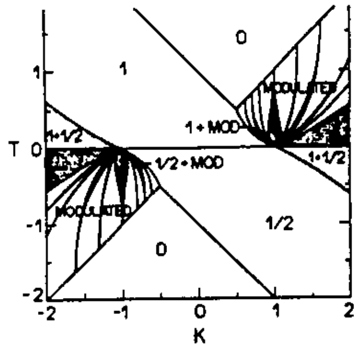

Figure 2: Global phase diagram in the $(T, \kappa, 0)$ plane.

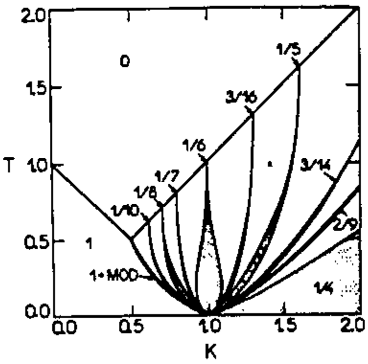

Figure 3: Main periodic phases in the first quadrant. The diagram on the third quadrant is symmetric to this one, with a serie $(V_1, V_2, V_3, V_4 \ldots)$ being changed to $(V_1, -V_2, V_3, -V_4, \ldots)$.

a two-cycles and the more complicated phase 2/9 is depicted as an example (fig. 4). Quasiperiodic phases and other periodic phases lie between the grey regions. Details concerning the determination of these lines are given in [13].

In the regions $1 + 1/2$ a fixed point and a two-cycle coexists. More interesting is the modulated-fixed point coexistence region (1+MOD) where periodic, quasiperiodic and chaotic attractors coexists [12, 13]. The same occurs at the coexistence region 1/2+MOD.

In figures 5, 6 and 7 we show diagrams in the plane T vs H, which are the most interesting for our neuron modelling purpose since they give the neuron response to external (constant) input I. The stability lines for the 0-phase are given by

$$H_c^{\pm} = (p-1)M_c^{\pm} + T \tanh^{-1} M_c^{\pm} \qquad (8)$$

with

$$M_c^{\pm} = \pm\sqrt{1 - \frac{T}{1-\kappa}}, \quad (p < 1/2) \qquad (9)$$

$$M_c^{\pm} = \pm\sqrt{1 - \frac{T}{\kappa}}, \quad (p > 1/2) \qquad (10)$$

Five general types of behaviour are encountered in these diagrams. For example, a neuron with sigmoid transfer function ($c_1 = c_2 = 1/2$) and delayed self-coupling $\kappa = 0.7$ (fig.6), with gain and threshold situated at point A will represent a formal neuron with resting near zero activity which, under external input, will show graded response. Neuron B shows, at intermediate external input level, oscillatory behaviour. Neuron C is a natural oscillator, a pacemaker with periodic or quasiperiodic behaviour even without external input. Neuron D has a very interesting 'bistable' behaviour. It has two coexisting attractors, one with low activity and other with high activity. An external instantaneous perturbation can put the neuron in the active state (figure 8). Neuron E is bistable at intermediate levels of external input I. Behaviours of type B and D remember some findings on thalamic neurons [14].

Detailed diagrams for the modulated regions with different values of κ are given in ref. [13]. As an example of the complex nature of these regions we show the diagram for $\kappa = 1$ (figure 9). In the coexistence region 0+MOD (inferior region of the bubble) we can also find chaotic attractors like the presented in figure 10.

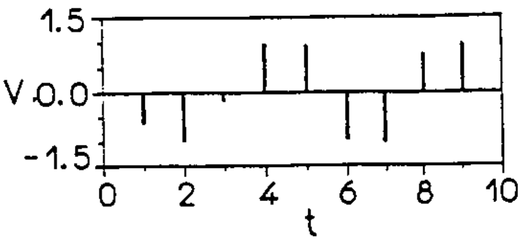

Figure 4: Example of a 2/9 phase. The series repeat each interval of 9 points, but there are 2 oscillations in the interval. The 'average wave lenght' is $\lambda = 9/2$.

4 Outlook and Conclusions

How does a network of DPs relates to the attractor (Hopfield-like) networks studied by the statistical physicists? Well, the most studied network [9, 10] is a fully connected set of N formal neurons without self-couplings and thresholds, with the (parallel) neuron dynamics given

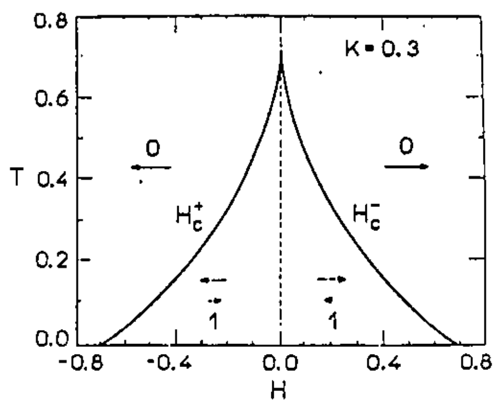

Figure 5: Stability limits for the 0 phase for $\kappa = 0.3$. In the region 1 coexists a phase parallel to H and other with smaller amplitude antiparallel to H.

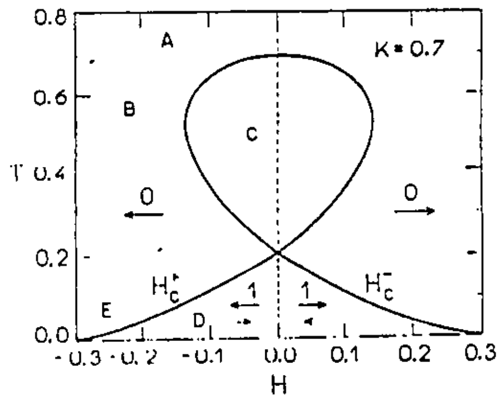

Figure 6: $\kappa = 0.7$: Modulated phases appear inside the bubble. A, B, C, D and E represent neuron models with qualitatively different dynamical responses.

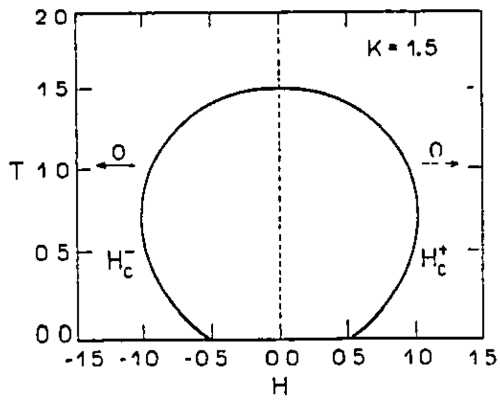

Figure 7: $\kappa = 1.5$. Modulated phases appear inside the bubble. There is no phase 1 for $\kappa > 1$.

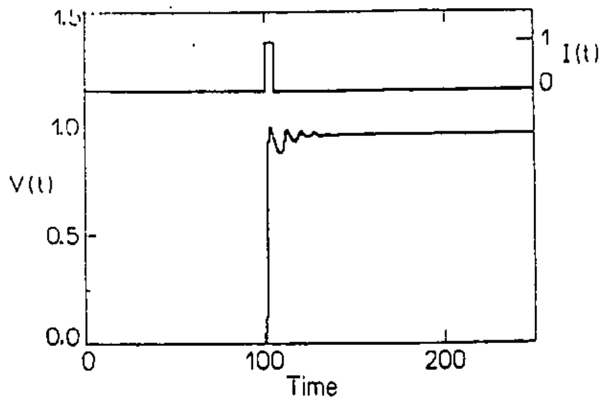

Figure 8: Plots of the activity $V(t)$ and external input $I(t)$ for a neuron of type D.

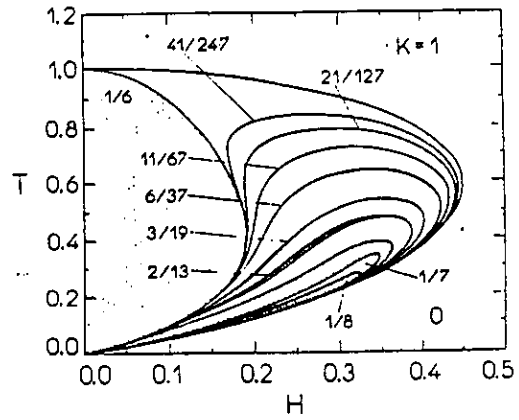

Figure 9: T vs H phase diagram for $\kappa = 1.0$

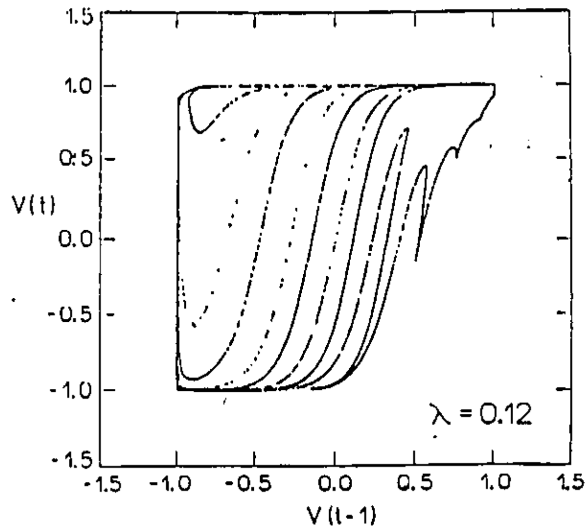

Figure 10: Strange attractor with largest Lyapunov exponent $\lambda_1 = 0.12$ found at the point $(T = 0.15, \kappa = 1, H = 0.235)$.

by

$$V_i(t) = \tanh\left[\gamma \sum_{j=1}^{N} J_{ij} V_j(t-1)\right] \quad (11)$$

with $J_{ii} = 0$. A phase diagram showing the recuperation (content-adressable memory), paramagnetic and spin glass phases has been obtained by Shiino e Fukai [10] using the replica method. This phase diagram is very similar to the obtained by Amit et. al. [1] for the Hopfield network composed of McCulloch-Pitts neurons. The network presents only fixed points to be indentified with stored memories.

A network with self-couplings $J_{ii} = W_{1i}$ has also been considered, the neuron dynamics given by

$$V_i(t) = tanh\left[\gamma\left(\sum_{j=1,j\neq i}^{N} J_{ij} V_j(t-1) + W_{1i} V_i(t-1)\right)\right] \quad (12)$$

A condition of stability for the fixed point phase was determined by Marcus and Westervelt [9]. If this condition is not satisfied a limit cycle of period two appear.

A network of 2-D dynamical perceptrons will have the dynamics given by

$$V_i(t) = tanh\left[\gamma\left(\sum_{j=1,j\neq i}^{N} J_{ij} V_j(t-1) + W_{1i} V_i(t-1) + W_{2i} V_i(t-2)\right)\right] \quad (13)$$

This network cannot be analysed by the usual statistical mechanics approach, being a globally coupled maps system of the type studied (almost numerically) by the dynamical systems comunity. The 'easy' ferromagnetic case with all $J_{ij} = J > 0$ should be studied first, where global synchronization phenomena may appear. It seems also interesting to study a system with well chosen couplings (say Hebb synapses) designed to store various synchronized attractors. The analysis of such systems is a future enterprise.

In conclusion, the phase diagrams in the (T, κ, H)–space represent all the time series implementable by a two input single-layer perceptron. We think that the richness of dynamical behaviours encountered has pedagogical value since gives us some feeling about the much more varied time series implementable by more complex networks. To our knowledge these are the first phase diagrams in coupling space for the behaviour of a feedforward neural network turned a dynamical system.

Our proposal is to represent neurons by these two-parameter *dynamical perceptrons*. By this slight extension of the graded response formal neuron we can model qualitatively various dynamical behaviours of biological neurons. Networks of such discret oscillator neurons enables the study of collective synchronization phenomena, which will be reported elsewhere.

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