Is Kohonen under Nyquist rules?

L.H.A. Monteiro, L. Zerbinatti and J.G. Chaui-Berlinck

Abstract— Numerical experiments show that the organization of the output layer in Kohonen networks is subjected to the Nyquist theorem. This result was inferred by observing how such an organization depends on the sampling rate chosen for collecting the electrocardiograms used as input signals.

Index Terms— Electrocardiogram, Kohonen network, Nyquist theorem, sampling rate

I. Introduction

The self-organizing map, known as Kohonen neural network[1], uses an unsupervised learning algorithm in order to translate the similarities of the input data into distance relationships among the neurons composing its output layer. Thus, after appropriately training the network, two input data with similar statistical features stimulate either the same neuron or neighboring neurons.

The learning algorithm is given by:

$$w_{ij}(t+1) = w_{ij}(t) + \alpha(t)h(t)[x_j(t) - w_{ij}(t)] \quad \text{if } i \in V(i^*)$$

$$w_{ij}(t+1) = w_{ij}(t) \quad \text{if } i \notin V(i^*)$$

The integer number j labels the position of a neuron in the one-dimensional input array. Each neuron in this array is connected with all neurons of the output layer by synaptic weights w_{ij}, where ij expresses the position in the output matrix. The value of the input corresponding to the neuron j is given by x_j. Thus, the vector $\vec{x}(t)$ represents the complete input at the learning step t. For an input $\vec{x}$, the output neurons compete among themselves for being activated. The winner is the one presenting the maximum value of the dot product $\vec{x}.\vec{w}_i$ and it becomes labeled in this step by i^*. Such a winning neuron defines a neighborhood $V(i^*)$ of radius r centered around it. At each step t, the weights of all neurons pertaining to $V(i^*)$ are updated according to the expressions above; the weights of the neurons outside $V(i^*)$ are not altered. This adaptation rule modifies the weight vector of i^* to more closely resemble the input vector that just stimulated it. And the weight vectors of the other neurons in $V(i^*)$ are modified in order to be stimulated by similar vectors in the following steps, leading to the formation of a topographic map of the input data. The neighborhood function h attains its maximum value at i^* and decays along the lateral distance of i^*.

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The parameter α is called learning-rate factor and it is usually taken in the range $0 < \alpha < 1$. The values of r and α decrease as the learning process progresses.

In this paper, some computational properties resulting from the organization of the neurons in the output layer is investigated by using sampled electrocardiogram record (ECG). An ECG shows the electrical activity of the heart, which normally precedes its mechanical activity[2]. This signal expresses the temporal variation of the potential difference between two points on the body surface.

We performed experiments with the Kohonen network where the inputs $\vec{x}$ were sampled ECGs. Such a network have been used for classifying ECGs[3-8]. Here our aim was to find how the output layer of the trained network is altered as a function of the ECG sampling rate. Nyquist theorem[9] states that the sampling rate must be equal to, or greater than, twice the highest frequency component of the analog signal in order to such a signal be accurately reconstructed from the samples taken at equal time intervals. Is the output layer of the Kohonen network qualitatively changed around the minimum sampling rate established by Nyquist?

II. Experiments

We recorded ECGs from four adult individuals (a, b, c, d) at a sampling rate of 500Hz. Four different conditions were imposed to the subjects: lying down-breathing normally; lying down-tachypnea; lying down-breath holding; exercising. The four time series of each individual were used for training the network during 1000 adaptation steps. At each step, a series was randomly selected and it was considered the input signal. We chose: dimension of the input array = 1000; dimensions of the output matrix = 4×4; initial learning-rate factor $\alpha(0) = 0.1$; initial neighborhood radius $r(0) = 4$. The begining of the P-wave was taken as the first point of each array. At each 100 steps, the learning-rate factor was reduced of 0.01, and at each 250 steps the neighborhood radius was diminished of 1. The function h linearly decreases with r and its maximum value holds 1. The connection strengths were randomly initialized. For each individual, the network was trained 10 times; and, for each one, a characteristic pattern in the output layer was formed in the 10 training processes. This typical pattern was qualitatively always the same, but it was different from individual to individual.

Then, we took the ECG signals and read them in such a way that they would correspond to a signal sampled at rates of 250 Hz (500/2), 167 Hz (500/3), 125 Hz (500/4) and so on, up to 10 Hz (500/50). We trained the network 10 times for each one of these "sampling rates" and noticed that the characteristic patterns (obtained in

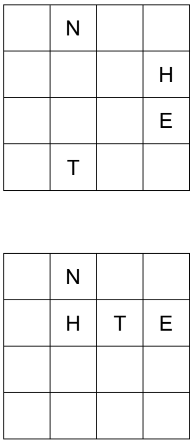

Fig. 1. Output layers of the individual a for 500 Hz (top) and 38 Hz (bottom). His output layer is not qualitatively changed for frequencies above 38 Hz (N: lying down-breathing normally; T: lying down-tachypnea; H: lying down-breath holding; E: exercising).

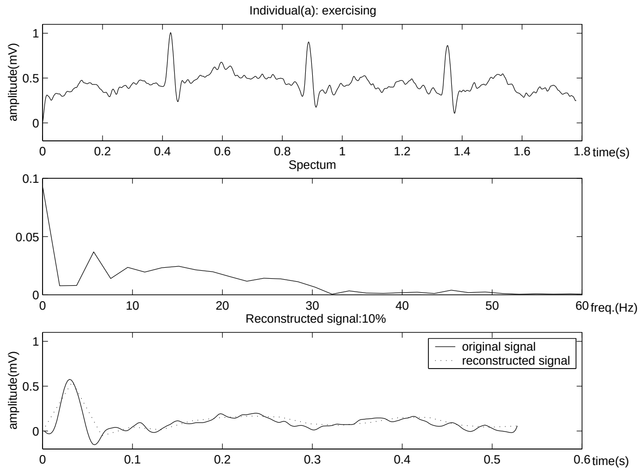

Fig. 2. Top: the original signal (sampled at 500 Hz) for the individual a in exercising. Middle: the corresponding spectrum. Bottom: one period of the original signal and the reconstructed signal by using the criterion of 10%.

the rate of 500 Hz) of the four individuals became altered below the following critical rates: *a*) 38 Hz (500/13), *b*) 45 Hz (500/11), *c*) 36 Hz (500/14), and *d*) 50 Hz (500/10). An example is shown in the Fig. 1, where the output layers of the individual a are presented for the sampling rates of 500 Hz and 38 Hz (his critical rate). Notice that by reducing the sampling rate, the correlation between successive samples of the ECG signal, which are the components of the input vector, is also reduced. Thus, the ECG signals tend to become similar to each other. Consequently, the Kohonen map loses its discriminative ability, clustering the signals very close to each other, as can be seen in Fig. 1.

There is a direct relationship between the coefficients of the Fourier series of a continuous-time signal and the coefficients of the Fourier series of such a signal sampled in a given frequency[9]. Thus, from the discrete-time series (the ECGs sampled at 500 Hz), we can determine the amplitude corresponding to each frequency appearing in the time-continuous Fourier series and plot its spectrum.

The spectrum of a signal provides a picture of its frequency composition. We found that the spectra of all 16 signals present a peak in the range of 1-10 Hz and decrease with increasing the frequency. Around 50-60 Hz, all spectra vanish. Fig. 2 presents an example.

We verified that each ECG waveform of each individual can be reconstructed with a good approximation by taking into account the frequencies appearing in the spectra up to the value corresponding to 10% of the peak amplitude (see Fig. 2). For the four individuals, we determined that the maximum frequencies with amplitude corresponding to 10% of the peak amplitude are: *a*) 20$\pm$2 Hz, *b*) 21$\pm$3 Hz, *c*) 19 ± 4 Hz, *d*) 21 ± 3 Hz. These values indicate the mean maximum frequency and its standard deviation concerning the signals recorded in those four different conditions.

Truncation criteria of 1% and 10% lead to reconstructed signals that are quite alike. Consider the expression $d_{10\%} = \sum_i (x_i - y_i)^2$, where x_i are the points of the original signal and y_i are the points of the reconstructed one by using the criterion of 10%. Thus, $d_{10\%}$ measures the "Euclidean distance" between these two signals. A corresponding expression can be defined for $d_{1\%}$. We calculated $d_{10\%}/d_{1\%}$ for the 16 ECGs and found that the average value of this quotient holds 0.93$\pm$0.04. Therefore, both criteria produce similar reconstructed signals.

By dividing each critical frequency (where the characteristic patterns in the output layer disappear) by its respective mean maximum frequency (related to the criterion of 10%), we obtained: *a*) 1.9 ± 0.2, *b*) 2.1 ± 0.3, *c*) 1.9 ± 0.3, *d*) 2.4 ± 0.3. Notice that the organization of the output layer in the Kohonen network is altered when the sampling rate is approximately two times the maximum frequency composing the signal. This result is according to the Nyquist theorem. When a truncation criterion of 1% is adopted, we found that the results of such divisions are: *a*) 1.2 ± 0.2, *b*) 1.5 ± 0.1, *c*) 1.0 ± 0.2, *d*) 1.3 ± 0.1. That is, the frequency related to the modification of the output layer would be approximately the maximum frequency presenting in the reconstructed signal.

III. Conclusion

The sampling rate determines the number of points constituting the time series. The higher the value of this rate, the higher the memory space used for storing the data. Here we showed that if we intend to employ these data for training a Kohonen network, then we can choose the sampling rate between once and twice the value of the highest frequency component, a result that is supported by the Nyquist theorem. Higher rates imply higher processing times for training the network without improving the quality of the final topographic map.

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