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Stock Market Simulation for Volatility Analysis Inspired on Ideal Gas: an Intelligent Agent Approach

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Abstract – This paper proposes a simulation environment for stock market analysis that uses intelligent agents. The behavior of the environment is defined by the ideal gas theory and the idea is to analyze the fluctuation of the stock markets and also the distribution of the gains and losses of the agents. The movement of the market can be estimated by a measure called volatility, which is defined by the difference between two stock prices in distinct periods. It characterizes the sensibility of a market change in the world economy. Thus, the contributions of this paper are: i) it is proposed a simulation framework of the stock market dynamics based on intelligent agents; ii) the volatility dynamics of the financial world indexes is analyzed; iii) a relationship between the volatility of the markets, the distribution of gains and losses of the agents and the coefficient of the exponential function based on the ideal gas theory of Maxwell-Boltzmann is proposed. In the experimental study, fifteen world market indexes were chosen to guide the simulation of the stock prices.

Keywords – Volatility Analysis, agents, Maxwell-Boltzmann theory and probability density function.

1. INTRODUCTION

The classical branch of science that aims to study and analyze the time series is Statistics. In this field, Box & Jenkins (ARIMA models) [1] is one of the most popular statistical techniques used to describe the behavior of time series. These statistical models aim to capture temporal patterns from the series under observation. When the objective is to understand the stock markets economic phenomenons, generally the specialists analyze the patterns contained in the return series. This analysis is important because quantifies loss or gain of a investment by the market movement. In other words, a significant market movement can lead to considerable difference between the price magnitude of a share in a short time. This market movement is called the volatility. Generally, a higher stock price fluctuation leads a higher volatility.

The volatility [2] can be defined as a statistical measure that quantifies the variation of returns of a share price or a financial index over time. There are several forms of estimate its value [2, 3]: mean of differences, variance or standard deviation between returns, historical volatility, implied volatility or more sophisticated models, such as exponentially weighted moving average (EWMA) used by RiskMetrics and GARCH process [1].

However, these analysis are not restricted to economists and statisticians. As the stock market is a complex system, computer scientists, mathematicians, engineers and physicists are applying new approaches based on statistical physics, nonlinear dynamics, statistical finance and stochastic processes to analyze the market behavior in order to obtain new insights or explanations about about its dynamics. Econophysics is the name this interdisciplinary research field that, through these new models try to understand the stock markets dynamics.

In Econophysics, traditionally, the probability density function (pdf) of the return series is used to analyze the stock market dynamics. The pdf is applied to determine the relationship between high volatilities and low volatilities. This analysis can be used to understand the markets behavior and to provide new insights about the economic fluctuations, high valorization and crashes in the value of stock markets, as well as, temporal evolution of money [2, 4–6].

The pdf of returns has been widely studied and many approaches for its analysis were developed. Mandelbrot [7], Fama [8] and Stanley [9] showed that distributions of returns can be fitted by a symmetric Lvy stable law. Other analysis showed that the pdf of the returns presents a Lvy distribution in the central part and exhibits a Gaussian behavior in the remaining parts, following the central limit theorem [9]. Cont *et al.* [10] proposed the fitting with exponentially truncated stable distributions and Longin [11] studied empirically the extreme events in returns with a Frchet distribution. Laherrere and Sornette [12] adjusted the distributions of stock returns by the Stretched-Exponential (SE) law. In last years, Queiros *et al.* [13] proposed the use of q-Gaussian distribution to fit financial data. Podobnik *et al.* [14, 15] analyzed the prices of stocks that comprise the Nasdaq and New York Stock Exchange. In these works, the pdf of aggregated returns are fitted by a double exponential in the central region.

Matia *et al.* [3] suggested that "hot" markets and "cold" markets have different behaviors. The volatility pdf for Indian stocks ("hot" market) is well fitted by an exponential law, on the other hand, American stocks ("cold" market) is fitted by a power law for the tail distribution. Thus, their work indicates that developing markets and developed markets can be fitted by different approaches. Although, in a recent paper, Mattos Neto *et al.* [16] analyzed seventeen world indexes and found that the exponential fitting adhered to data better than the classical approach of Econophysics, the power laws. The power laws are shown in Equation 1, where a and k are respectively constant of proportionality and exponent. The power laws are quite widespread in the Econophysics literature [4, 6] and are often used in various analysis, including scaling properties, relation between large and small volatilities and observation of extreme events.

$$y = ax^k \tag{1}$$

The Equation 2 shows the exponential function,

$$P(x) = a.e^{(-B.x)} \tag{2}$$

where the coefficients a and B are constants: a is the initial amplitude ($x = 0$) and B is a decay rate.

The objective of this paper is to create an environment of negotiation of stock markets based on intelligent agents. The dynamics to the agents follows the movement of an ideal gas, in which, the interaction between the particles (agents), if it exists, is weak. Evidences that the market could be modeled by this approach was showed by Mattos Neto *et al.* [16, 17]. They used a small time window (two years) [16] and artificial time series based on the Random Walk model [17]. In this paper, fifteen real world indices with a time window of 15 years are analyzed.

From these analysis, it is possible to verify if the analogy with an ideal gas can be used to describe the dynamics of the market in large time windows. Through this result, the idea is to relate the different types of markets based on the gains and losses of their agents. To achieve that aim, a relationship among the temperature of the markets, volatility and the gains and losses of the agents is established.

This paper is organized as follows. The Section 2 explains the base for the simulation, ideal gas theory, to analyze the stock markets and describes the environment simulated, the dynamics of intelligent agents and the methodology of the proposed analysis. In Section 3 evidences and an analogy between the dynamics of particle movements of an ideal gas, described by the Maxwell-Boltzmann Distribution, and the dynamics of stock markets and agents is presented in Section 3. And in Section 4 is discussed the final remarks.

2. METHODOLOGY

A stock market is a public entity for the trading of company stock (shares) and derivatives at an agreed price; these are securities listed on a stock exchange as well as those only traded privately. The market can be seen as a network of economic transactions, not a physical facility. In recent years, the most of operations in market are realized on line by intelligent agents [18].

The approaches based on agents to financial analysis has grown into an important research field for developing and understanding the complex patterns and phenomenas that are observed in economic systems. The agent-based computational economics [18–20] and the micro-simulation approaches have been proposed, emphasizing the need to represent traders as individuals and to study the way macro features emerge from individual interactions. In this new interpretation the concept of complex systems can be used.

A complex system is any process composed for many elements and that has many relations among them, so the global behavior depends on each component and its behavior depends on the behavior of others components of the system. The stock market is an example of a complex system [2]. However, simulate all variables of market stock is very complicated and costly, then a simple simulation was developed to analyze the characteristics of market. In [16] an analogy using a particle system (ideal gas) was used to explain the macro behavior of stock markets. Thus, this analogy could be used to describe the dynamics of differents markets of world with its shares and agents.

The simulated stock market is based in ideal gas theory [21]. An ideal gas is a theoretical gas composed of a set of particles that move randomly. These particles weakly interacting among them, or not interacting among them. The ideal gas concept is useful because it obeys the ideal gas law, a simplified equation of state, and is amenable to analysis under statistical mechanics. In economic theory, there are two basic ideas about the dynamic of stock markets. The first is the random walk hypothesis, this financial theory affirms that the market evolves according to a random walk model. Thus, the behavior of market cannot be predicted [22, 23]. The second theory, called of non-random walk hypotheses, affirms that the stock market is predictable to some degree [24, 25].

Assuming that the stock markets are formed by particles (agents), on these particles are not interacting with each other (random walk hypothesis) or are interacting weakly (non-random walk hypothesis). Therefore for both cases, the market can be seen as an ideal gas [16]. From this analogy, the financial values of the stocks can be viewed like energy of the particles.

Following this hypothesis was generated a simulated environment, where the agents do not interact with the others, only with the market as can be seen in Figure 1. The negotiations (orders of buy and sell) are done only between the agents and the market, there is not interaction among the agents. The actions of financial agents do not influence the actions of others financial agents, neither the fluctuation of stock prices. The idea is to generate a market, where the interactions are realized only between each financial agent and the market, or stocks.

The two environment are initialized: agents and stocks. Each agent has a name, an initial capital, one stock in its possession and one stock that it desire to buy (target stock). The characteristics of financial agents are generated randomly: the initial capital is generated by a gaussian distribution ($initial\ capital \approx N(10000, 100)$) and stock in possession and target stock are chosen randomly. The stocks follows the fluctuation of a real index market. Each stock initiates the simulation with a name, its initial price and a standard deviation. The name serves only to identify each stock and it is generated sequentially and the initial price

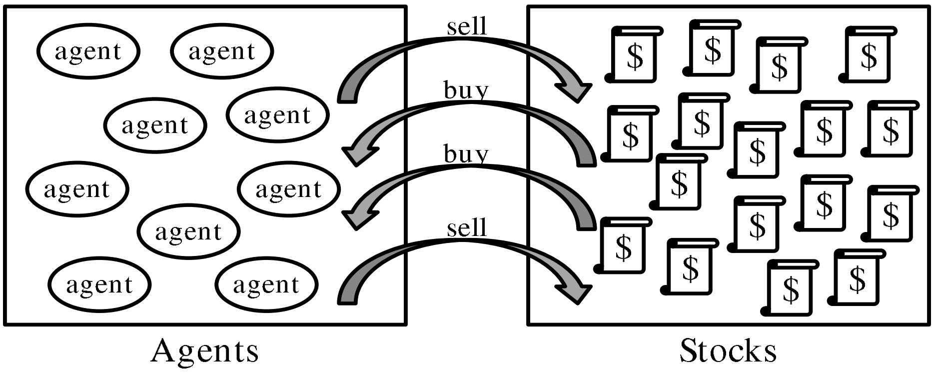


Figura 1: Simulation of market.

is always a positive number generated randomly from index market by a gaussian distribution ($price \approx N(index, 10)$). In each iteration, that corresponds to one day, the financial agents buy and sell financial securities randomly with probability of 90%, however only one operation is allowed for iteration (or per day). After of all negotiations the prices of stocks are updated based in index record of actual iteration.

The world indexes chosen are daily records in the period of fifteen years ($1995 - 2010$), accounting for approximately 3700 days. The series were: United States of America (Dow Jones, Nasdaq and S&P500), England (FTSE100), Japan (Nikkei 225), Germany (Dax 30), French (CAC 40), Spain (Ibex 35), Singapure (STI), Mexico (IPC), Hong Kong (Hang Seng), Malaysia (KLSE), Brazil (Ibovespa), Austrian (ATX) and Switzerland (SMI).

In the literature of Econophysics, there are several approaches to analyze the pdf of return series [12,13,26], however two are normally used, exponential functions and power laws. The idea is to generate an artificial environment based in real indexes and analyze the characteristics of market in a large time window. Developed markets normally are characterized by lower volatilities and developing markets are characterized by high volatilities.

Below are describes the variables of the simulation:

- Number of agents $= 500$;
- Number of stocks $= 5000$;
- Iterations $\cong 3700$;
- Probability of buy $= 0.9$;
- Probability of sell $= 0.9$.

For each index was design the following steps:

1. The return series is defined for each series as

$$g(t) = \frac{(\log Z(t + \Delta t) - \log Z(t))}{\delta}, \tag{3}$$

where $\Delta t = 1$ day, $Z(t)$ is the series value at time t, and δ is the standard deviation of $(\log S(t + \Delta t) - \log S(t))$. This formula [3] describes how the return series are constructed from index series;

2. For each index, the volatility [27] described in Table 1 was calculated as

$$V_t(t) = \frac{1}{n} \sum_{t'=t}^{t+n-1} g(t), \tag{4}$$

with $n = N$, where N is the total number of time series observations;

3. The probability density function of the return series is estimated;

4. The comparison between the power law adjust and the exponential functions adjust is done based on Trust Region algorithm [28]. Mean of Squared Errors (MSE) is used to evaluate the results.

The data (pdf of the volatility of the series) was used and the fitting procedure was done based on Trust Region (TR) algorithm [28]. The TR algorithm, also known as restricted step method, searches for the region which solves the minimization problem in question. For this it uses a model function (often a quadratic one). When the TR algorithm finds a model according to the objective function, the region is expanded, trying to find new promising regions to solve the problem, conversely, if the adjustment is poor, the region is contracted, and the algorithm searches for other regions that can solve the problem. The Trust Region provide numerical solutions, using a linear search method to the problem of function minimization.

Tabela 1: Fitting errors (MSE) using power law and exponential functions

RW Model	Volatilities of series	Trust Region		Exponential Coeff. (B)	Pearson Coefficient
		Power law	Exponential		
Nikkei	1.030	0.00018	0.00024	0.981	1.3341
CAC 40	1.009	0.00021	0.00009	1.020	1.3905
Ibex 35	1.005	0.00020	0.00006	1.016	1.3446
DAX 30	0.993	0.00020	0.00008	1.003	1.3207
IPC	0.988	0.00035	0.00011	1.036	1.3241
Nasdaq	0.974	0.00021	0.00006	1.024	1.3469
SMI	0.971	0.00021	0.00005	1.049	1.2876
FTSE 100	0.963	0.00021	0.00014	1.056	1.3432
Ibovespa	0.963	0.00020	0.00012	1.075	1.3022
DJIA	0.937	0.00014	0.00036	1.077	1.3173
STI	0.933	0.00020	0.00006	1.087	1.2495
S&P 500	0.919	0.00026	0.00009	1.098	1.237
Hang Seng	0.903	0.00020	0.00010	1.089	1.2654
ATX	0.899	0.00018	0.00010	1.151	1.2836
KLSE	0.706	0.00021	0.00012	1.480	1.0009

3. ANALYSIS

As a first step, the exponential function and the power laws were used to fitting the data using the algorithm TR. The results of simulations with the agents are shown in the Table 1. The results are in descending order of the volatilities. It can be seen that the exponential function obtained a better fit than the power law for most of indexes.

Figure 2 shows examples of the fitting generated by the exponential function and by the power law, where the black points are the experimental observations and the gray lines are the fits. The examples are the indexes of Spain (higher volatility) and Singapore (lower volatility). The exponential function adheres better to the probability density function of the series because it is capable of doing a good fitting for most data of the volatility pdf, including the end of the tail. The tail is the region of the pdf that has a more instable behavior. A good fitting in this part is important because there is a considerable concentration of points and in this zone is where the large gains and large losses are obtained.

This instable behavior at the end of the tail occurs due to the high values in the return series; these are the cases where the investor has higher gains or higher losses, which can represent a possible crash [4]. Therefore, it is very important to understand the dynamics of this part of the return series, even knowing that it occurs with a low probability.

From the observation of the adjustment of pdf, a comparison with the theory of ideal gas can be established. A reasonable association can be done between volatility and temperature of the market (a coefficient of agitation). If a market is more agitated (or "hotter"), it has higher volatility, otherwise, if the market is not agitated, it has lower volatility (or "colder"). Temperature implies thermal energy; therefore, if the market is observed from the point of view of an ideal gas, the process resembles an ideal gas guided by a Maxwell-Boltzmann Distribution [21]. Based on this analogy, the markets would be characterized by a given temperature (or volatility).

The theory of an ideal gas, more precisely the Maxwell-Boltzmann Distribution [21], describes the probability function of the particle speed or energy, where the particles do not constantly interact with each other but move freely between short collisions.

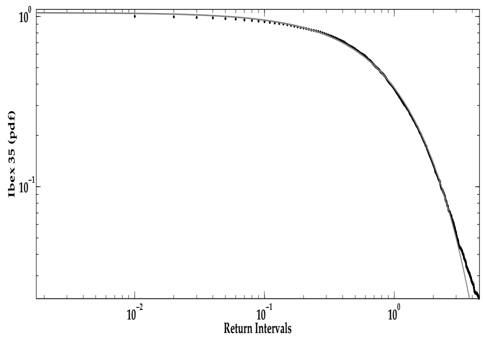
This statistics is used to describe the distribution of particles over various energy states in thermal equilibrium, when the density of the particles is low and the temperature is high, discarding the quantum effects. This variation in the speed generates thermal energy, resulting in an increase of the agitation of the particles.

The theory of an ideal gas corroborates with the simulations created. The dynamics found in the pdf of indexes, using time series with large time window follows the Maxwell-Boltzmann Distribution. Thus, the stocks or companies shares that are negotiated by investors (financial intelligent agents) could be compared with particles of an ideal gas in the Maxwell-Boltzmann Distribution, like in simulation. In the experiments, the environment recreates the idea of an ideal gas, since there is not exchange of information between the agents, nor between the stocks of market and nor between agents and the market. With this in mind, the temperature of an ideal gas can be related to the volatility of the market. Thus, higher temperature leads to higher volatility and lower temperature of the economic system leads to lower volatility of the market. A relationship between Equation 5 (the Maxwell - Boltzmann statistics) and Equation 2 (exponential function) is given by Equation 6.

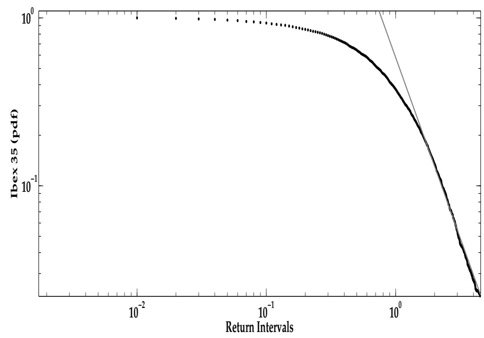
$$y = a.\exp^{-E/k_b.t} \tag{5}$$

$$B = \frac{1}{k_b.t} \tag{6}$$

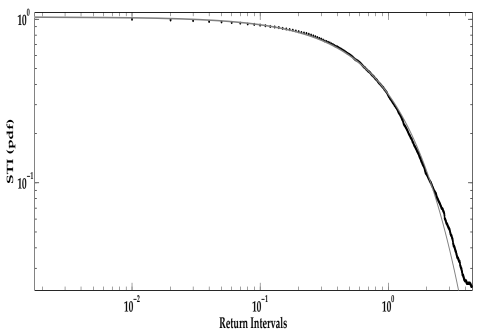
This theory can be used to explain the relationship found between the B coefficients of the exponential function and the volatility. The system temperature (Equation 6) can be extrapolated to the market system, as observed in Figure 3 and in Table 1:



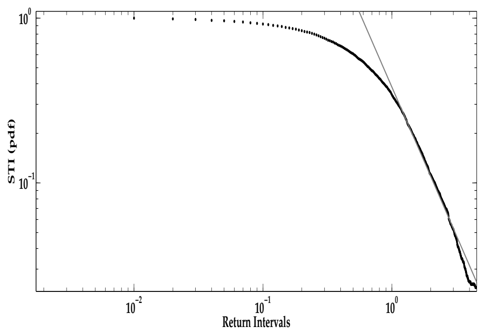
(a) Exponential function.



(b) Power law.



(c) Exponential function.



(d) Power law.

Figura 2: Comparison of fitting with power law and exponential function, using Trust Region algorithm in log-log scale. (a-b) Fitting to Ibex 35 index. (c-d) Fitting to STI index. The black line is the probability density function (pdf) of series and gray line is the fit line

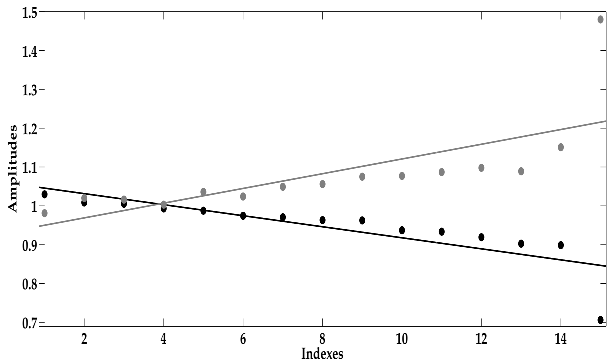


Figura 3: Comparison among the indexes volatilities and the B coefficients. In plot, the gray dots are the B coefficients and the black dots are the volatilities of the indexes. The black line and gray line are the linear fitting.

when the volatility decreases (the energy decreases or the temperature decreases), the B coefficient increases. Figure 4 shows that the product between B coefficient and volatility ($B \cdot Volatility$) tends to be a constant, as can be demonstrated by Equation 7,

$$B \propto \frac{1}{Volatility} \Rightarrow B \cdot Volatility = C \tag{7}$$

where C is the proportionality constant between the B coefficient and the volatility. The constant found in the experiments was $C = 1 \pm 0.02$.

These facts indicate that the Maxwell-Boltzmann theory can be really used to analyze the behavior of the pdf of the return series presented in this article. Thus, the market system and its components could be treated like a gas system [21]. In the experiments, Equation 7 confirms the association found between the exponential coefficients and the volatility. The B coefficient, or equivalently the volatility, can be used to quantify the financial risk of a given market over a specified time period, estimating of the instability/fluctuation of the markets. Thus, the degree of the market's fluctuation may be compared as the particle's energy, which can be seen as the temperature of market. In Equation 6, the same association can be seen: the B coefficient is inversely proportional to the temperature of the system. This is a valuable information which can be used to cluster different markets, to observe the characteristics in common or to predict the behavior of different markets, or of a group of markets.

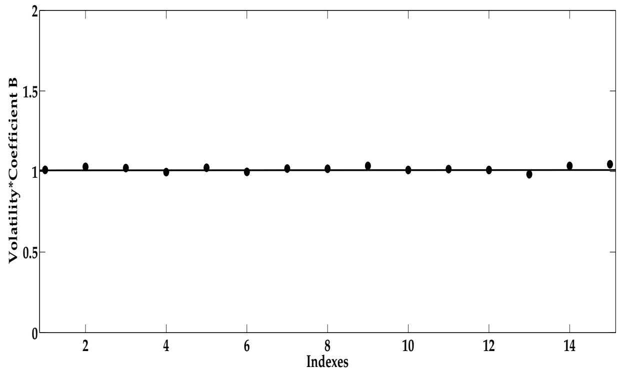


Figura 4: The product $B \cdot Volatility$ for each market. The solid line is the mean value of these dots.

From these tests, the dynamics of agents can be better analyzed, like the distribution of gain/loss of money by agent. In simulation, the agents do not have any information about the market, or about the investments of the other agents. These agents corresponds to people that invest without to make any study of market, or any type of forecasting based in calculus. These people invest only using their “emotional” side, or put the money in the stock market as long time investment.

To calculate the distribution of gain/loss of money per agents, a histogram for each market was constructed. The Pearson coefficient [29], used to estimate the asymmetry of histogram, was calculated for each series and its values are in Table 1 in descending order following the volatility. The Pearson coefficient for asymmetry applies in the case of the slightly asymmetrical distributions. It is determined through relating the difference between average and mode to the average square deviation. The Pearson coefficient is measured according the Equation 8,

$$A_s = (\bar{X} - Mo)/\delta \tag{8}$$

where A_s is the Pearson coefficient, $\bar{X}$ is the mean of data and Mo is the mode of data. How the analyzed data are return series, higher the value of Pearson coefficient, higher the asymmetry of the data. So more agents gain or lose money. The relationship among Pearson coefficients and B coefficients are showed (in crescent order of B) in Figure 5.

Figure 6 shows the histograms of distribution of money of Nikkei 225 and KLSE indexes. Comparing the histograms in Figure 6 may be noted that for histogram of smaller volatility (KLSE), the agents gain/loss less money than the histogram of higher volatility (Nikkei 225). These measures show that when the volatility is small, the gain/loss of investors are around the average of return. When the volatility is high, there is a bigger spread and the probability of higher gain/loss of the agents is bigger. These results corroborate with the Maxwell-Boltzmann theory described in this paper. The agents that too are particles gain/loss more energy, according with energy of system.

Then, there is a relationship: between the volatility and the B coefficient of exponential function, between the volatility and Pearson coefficient and the probability of gain/loss of agents and finally, the relationship among B coefficient and the Pearson coefficient.

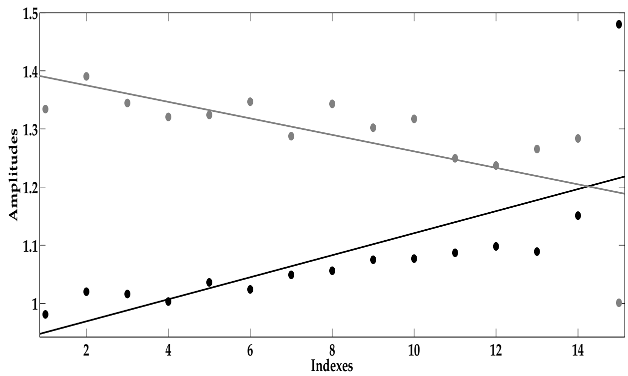
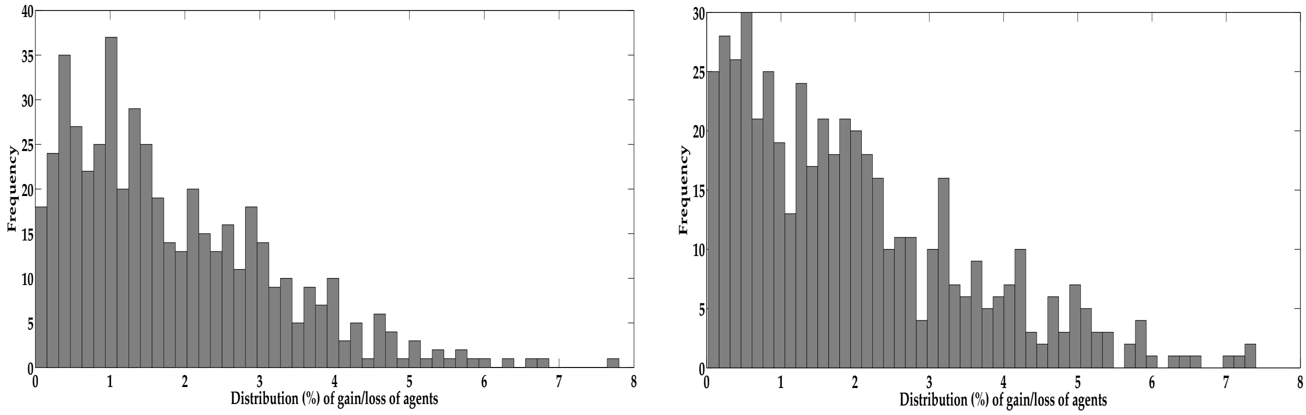


Figura 5: Comparison between the Pearson Coefficient and the B coefficient in crescent order of volatility. In plot, the gray dots are the Pearson Coefficients and the black dots are the B coefficients of indexes.

4 CONCLUSIONS

In this paper was presented a simulation with 15 world markets, representing developed and developing economies. The simulations were based in the assumption that the dynamics of stock market follows the ideal gas theory [16]. The proposed



(a) Histogram of Nikkei 225. (b) Histogram of KLSE.

Figura 6: Comparison of the histograms of distribution (gain/loss) of agents. (a) Distribution of money of the agents to Nikkei 225 index. (b) Distribution of money of the agents to KLSE index.

environment was based in [17], however in this paper, the artificial stock prices are guided by indexes of real markets. In first step, the simulation was recreated to simulate an artificial environment, where the agents interact only with market through of buy and sell orders. The agents do not exchange information, either between them or with the market. And the stocks receive information only of market index.

Then, two classical approaches of Econophysics were used to describe the pdf of returns. Based on the results achieved for these real indexes, the exponential function adhered better to data of probability density function (pdf) of the real markets than the power laws. Since the exponential function fitted all regions of data, while that the power law only fitted the tail of data. After the exponential function adjustment, a relationship between the B coefficient of exponential function with volatility was found. This relationship confirms the evidences that an analogy could be used to explain the relationship between the volatility (temperature in the Maxwell-Boltzmann Distribution) of the markets and the B coefficients of exponential. This approach can bring insights about the modeling of the stock market volatility. Therefore this conjecture seems to be promissory to new modeling of dynamic market.

Thereby, in a finance system, the volatility can be seen as the temperature, when compared with an ideal gas system. The larger the market agitation, the higher the temperature of the markets and the higher the volatility of the series. These conclusions are supported by the results and strengthen the Maxwell-Boltzmann approach to model financial time series, validating the results and conclusions demonstrated in this paper. The B coefficient relates the frequency of occurrence of small values and high values in the return series, and therefore could be used to measure, or to quantify the movement of markets, as volatility. From that information, the investor can forecast the behavior of the market or can understand the association between high and small volatilities. This can be an interesting tool for financial market analysis.

Finally, another association can be established among B coefficient, or volatility and the distribution of gain/loss of agents. This behavior was measurement by Pearson coefficient, through the asymmetry of gains/losses of the agents. Higher the volatility of market, higher the asymmetry of the histograms of data and higher the energy of each agent. The energy of agent is determined by how much it won/lost money. Then, this result corroborates with the dynamics of the Maxwell-Boltzmann Distribution. A important detail is that the agents do not have any information about the market, so its dividends won or lost follow the market. This contribution may help in understanding of the stock markets and in analysis of agents performance, and from this assist in the elaboration of the strategy of trader.

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