

GENETIC ALGORITHMS APPLIED TO COMPUTER-GENERATED GREEN PUBLIC LIGHTING DESIGN

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Abstract – Lighting Design is a field of engineering that misses automatic approaches to help lighting designers. Genetic Algorithms (GA) are a widely used class of algorithm for search and optimization. This work presents the early results from applying GA for computer-generated public lighting design. Some simplifications are done in order to test the extent and efficacy of this methodology. The solution designed by GA, when compared to the edified one, shows a superior uniformity of illumination and a 10% of economy on power consumption.

Keywords – Computer-generated Design, Public Lighting Design, Genetic Algorithms.

1. INTRODUCTION

Lighting design has an extreme importance due to the high cost of implantation and maintenance of the illumination installations. Best illumination dimensioning is indispensable to achieve an adequate lighting and reduced electrical power consumption.

At the design procedure, the engineer is challenged to specify the best solution for a given problem. To choose the conjunction of technical characteristics in the definition process, it is necessary to evaluate the lower cost of installation, the best fixture height of the luminaires capable to provide sufficient luminance level according the standards, and at the same time it is necessary to guarantee that the final solution presents a good cost-benefit relationship considering the overall power consumption a lower index of maintenance and much more.

However, most of the current systems depends on the project sequence adopted by the engineer planner, and the final results depend on his/her experience. Furthermore, the engineer spends a lot of time trying to reduce costs and searching for better solutions, since no such features are provided by the computer.

A genetic algorithm (GA) is a search heuristic that mimics the process of natural evolution. This heuristic is routinely used to generate useful solutions to optimization and search problems. Genetic algorithms belong to the larger class of evolutionary algorithms (EA), which generate solutions to optimization problems using techniques inspired by natural evolution, such as inheritance, mutation, selection, and crossover [1].

This work proposes a Genetic Algorithm to optimize lighting design parameters, offering itself as a computer-generated design application for green public lighting design. It is derived from author's previous work [2]. Assuming a green design, we mean a design concerned with respecting standards, providing human comfort, and promoting the efficient use of energy. From all green criteria for lighting design, this first version starts with only two: the search for a better uniformly distributed illumination, and the choice for the solution weighted by the total power consumption. The resulting of this project will be integrated with a Computer Aided Design (CAD) application [3].

This work also presents the case of a lighting design requested for the Uberlandia airport (UDI) parking lot. The edified design was performed by an engineer, using several available tools to help him. The solution achieved by GA, when compared to the edified one, shows a superior uniformity of illumination and a 10% of economy on power consumption.

2. BACKGROUND

2.1 Illuminance evaluation

The directed lighting method (or point-to-point method) is widely used for illumination calculation in outdoor areas. With this method, the illuminance (the light incident on a surface) from one or more light sources is evaluated at a given point or even through a discretized area of interest. A light source is defined by a given luminaire. Luminaire, or light fixture, is a “complete lighting unit, consisting of one or more lamps (bulbs or tubes that emit light), along with the socket and other parts that hold the lamp in place and protect it, wiring that connects the lamp to a power source, and a reflector that helps direct and distribute the light”¹.

A luminaire distributes, filters or converts the light emitted by one or more lamps. Part of the luminous flux emitted by the lamps is absorbed by the luminaire and does not contribute to the ambient lighting. The flow balance is spent above and below a horizontal plane passing through the center of the luminaire. The beams of light irradiated directly on the working plane are

¹Luminaire. (2011). In *Encyclopædia Britannica*. Retrieved from <http://www.britannica.com/EBchecked/topic/1381794/luminaire>.

the main contributors to the luminance. For each luminaire, the photometric data used for calculating illuminance are available through their respective IES files, a file format maintained by the Illuminating Engineering Society and used by many existing softwares, e.g. [4].

Inverse square law states that the illuminance E (S.I. unit: lumen) at a point on a surface varies directly with the intensity I (S.I. unit: candela) of a point source and inversely as the square of the distance d (S.I. unit: meters) between the source and the point on the working plane [5]. The illuminance evaluation deals with geometric quantities shown in Figure 1 and is given by:

$$E_i(P_j) = \frac{I_i(\gamma, \psi) \cdot \cos\gamma}{d^2} \quad (1)$$

where $I_i(\gamma, \psi)$ is luminous intensity² of the light source i in the direction of the point P_j ; j is the index of a given point on the discretized working plane; γ is the angle between normal to the surface on point P_j and the direction of luminous intensity I_i ; and d is the Euclidean distance between the light source and the point P_j .

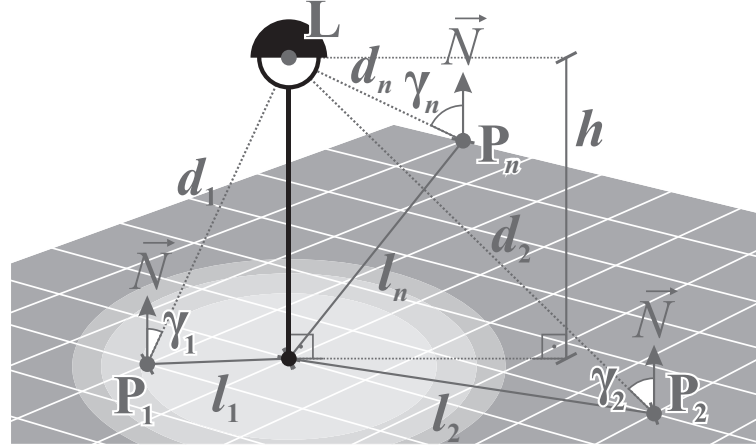


Figure 1: Guideline to geometric quantities.

Regarding Figure 1, \vec{N} is the normal to the plane; γ_i is the angle between \vec{N} and the segment $\overline{LP_i}$; h is the height of the lamp (pole); l_i is the horizontal distance between the light source (represented by point L) and the point P_i of observation; d_i is the Euclidean distance from the light source to the point P_i ; $i = 1, 2, \dots, n$; and n is the number of observation points in the discretized space constituted by the mesh nodes generated for the area of interest.

The total illuminance for a given point is evaluated by adding the contributions of each one of the light sources:

$$E_{total}(P_j) = \sum_{i=1}^N E_i \quad (2)$$

where: $E_i(P_j)$ is the illuminance contribution from the light source i to the point P_j ; N is the number of light sources.

2.2 Genetic Algorithms

Genetic algorithms are typically implemented for computer simulations where an optimization problem is specified. Members of a space of candidate solutions, named as individuals, are represented using abstract chromosomes. The GA consists of an iterative process that evolves a working set of individuals denominated as a population toward an objective function [1], also known as fitness function.

Solutions are evolved by using fixed length vector as chromosomes, usually binary vectors. Alternative structures have also been developed to able real number vectors or even symbolic ones. The evolutionary process of a GA is a simplified and stylized simulation of the biological version. The population of individuals are initially generated by random according to some probability distribution, usually the uniform distribution. The update of this population is then taken in successive steps denominated as generations. In each generation, multiple individuals are selected from the current population based upon some application of the fitness function with some random process. The new generation is bred using crossover between previous selected individuals, and then modified through random mutations to form a new population. The best individuals, based on the fitness function, could be kept in the new generation by a process named elitism.

In this work, a genetic algorithm is proposed to find the best combination of pole heights and luminaires, including their respective positions, as a method of computer-generated lighting design.

²Usually provided by photometric data file in IES format.

2.3 Work considerations

In order to validate the proposal of this work, i.e., to test if GA is able to perform computer-generated lighting design, some simplifications were taken:

1. Developed application is dedicated to outdoor (public spaces) lighting design; it means that reflectance contributions for the total illumination of the area are not considered.
2. The only difference between chosen lighting poles is their respective heights.
3. The luminaire position is considered to be exactly the lighting pole position, and can be installed only parallel to the ground (focus on the luminaire location point).
4. An unique hypothetical direct lighting luminaire was developed (100% of their output is directed downward), leaving no choice for different fixtures; it holds just one electric lamp at a time; exchanging the lamp is allowed, though.
5. Just 20 different lamps were taken into the project; luminous flux and wattage are their only considered features regarding the final solution.
6. The lowest set "A" (30 lx) is assumed for the public space as the optimal illuminance target, as declared in the table of illuminance categories [5], with a ± 10 lx tolerance.

2.3.1 Lighting Pole

Generic lighting poles are considered to this project, with heights of 4, 7, 10, or 12 meters. All of those are above ground heights. For simplifications, they are assumed to hold just one fixture.

2.3.2 Fixture

Photometric curves are representations for the distribution of the luminous intensity emitted from a source, usually found as polar diagrams. The represented space has a "pole" in the center of the luminous source, and a polar axis corresponding to the straight line exiting from the pole and perpendicular to the plane to be illuminated. Luminous intensity emitted from a source can be graphically represented with a segment which, starting from the center of the source, has a length proportional to the value of the intensity. Different fixtures can modify distributions in space of the luminous intensity produced by a given lamp. In polar photometric curves, the luminous intensity distribution from a given luminaire is registered toward different light propagation directions.

Manufacturers of luminaires usually provide the photometric curves related to their products presented in their catalogs. Some also provide through their website the related photometric data files in the IES format³. Using the photometric data from a specific installed luminaire, the illuminance of any point on the working plane can be calculated. Actually, those files provide values of luminous intensity in discrete angles increment. But it is possible to calculate a value for an intermediate angle using an interpolation criteria.

To avoid interpolations and as a part of the simplification efforts, the only type of fixture considered in this work is a hypothetical one, designed as a direct lighting luminaire (100% of its output is directed downward). The luminous intensity distribution takes the form of a sphere, with the luminaire located in the zenith. It can be described by a function, so its distribution is continuous. Looking through the planes parallel to the working plane, it has an isotropic luminous intensity distribution. In planes perpendicular to the working plane, it has the appearance of a circle. The IES-like photometric curve for this ideal luminaire is shown in Figure 2.

When integrating a given lamp to this hypothetical luminaire, its respective luminous intensity I , in candelas, are given in function of the observed direction:

$$I(\gamma, \psi) = \frac{\Phi}{4\pi} \cdot \left(\frac{1 + \cos 2\gamma}{\cos \gamma} \right) \quad (3)$$

where Φ , in lumens, is the luminous flux; γ is the elevation or vertical angle; and ψ is the horizontal angle. Note that I is independent of ψ , i.e., it is isotropic through the planes parallel to the illuminated area⁴.

The maximum luminous intensity is given in the direction of the luminaire focus, i.e., where $\gamma = 0^\circ$. On the working plane, it is assumed to be the point straightly below the luminaire (the exact location of the luminaire), because this hypothetical luminaire can be only installed parallel to the ground.

³IES (Illuminating Engineering Society) file format for the electronic transfer of luminaire component data, a industrial standard.

⁴To register the photometric curve normalized to the unit $cd/1000\text{ lm}$, just do $\frac{I}{1000\Phi}$.

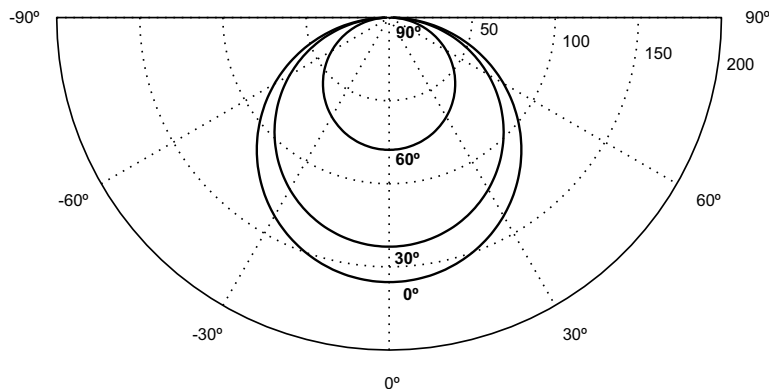


Figure 2: Photometric curve of the ideal luminaire, units in cd/1000 lm.

2.3.3 Lamps

To run our tests, 20 different lamps were included in the lamp database for the GA application. It's important to notice that, at any time, lamps can be included or excluded, at user's discretion. For this work, the chosen lamps are divided in four main groups:

- MASTER CosmoWhite CPO-TW: New-generation ceramic metal halide lamps for outdoor lighting with white light, using a new lamp base and clear quartz outer bulb, from Philips [6]. Application: Urban Public Lighting for city centers, residential areas and roads.
- MASTER CityWhite CDO-TT: Ceramic metal halide outdoor with clear tubular outer bulb, from Philips [7]. Application: City centers, shopping centers and pedestrian areas, residential areas, road lighting and floodlighting.
- SON: High Pressure Sodium lamp with opalized ovoid outer bulb, from Philips [8]. Application: Road and residential lighting, Decorative floodlighting, Commercial and industrial applications.
- Powerstar HQI-T: Metal halide with quartz technology, from OSRAM [9]. Application: Stadium lighting, Street lighting, Industrial and workshop lighting.

Table 1 presents the product name, manufacturer, the lamp wattage, and the luminous flux (both rated EM 25°C) for each lamp included in the database.

Table 1: Lamps database for outdoor lighting projects.

Product Name	Manufacturer	Lamp Wattage EM 25°C, Rated	Luminous Flux EM 25°C, Rated
CPO-TW 45W/728 PGZ12	Philips	45	4725
CPO-TW 60W/728 PGZ12	Philips	60	6800
CPO-TW 90W/728 PGZ12	Philips	90	10450
CPO-TW 140W/728 PGZ12	Philips	140	16500
CDO-TT 250W/828 E40	Philips	244	22500
CDO-TT Plus 70W/828 E27	Philips	73	7500
CDO-TT Plus 100W/828 E40	Philips	98	10700
CDO-TT Plus 250W/830 E40	Philips	250	28500
SON 50W/220 I E27	Philips	50	3400
SON 70W I E27 CL	Philips	70	6000
SON 100W/220 E40	Philips	100	8500
SON 150W/220 E40	Philips	147	14500
SON 250W/220 E40	Philips	250	27000
SON 400W/220 E40	Philips	400	48000
SON 1000W/220 E40	Philips	1000	130000
HQI-T 70W/NDL G12	OSRAM	75	5500
HQI-T 150W/NDL G12	OSRAM	150	13000
HQI-T 250W/D E40	OSRAM	250	20000
HQI-T 400W/D NAV-VG E40	OSRAM	420	32000
HQI-T 400W/D HQI-VG E40	OSRAM	360	25000

2.4 Proposed chromosome

One of the main problems initially presented in this study was to choose a proper representation of the chromosomes. The main disadvantage on regular chromosomes for GA is the fact they need to be of a fixed length. However, a lighting design problem has an undetermined optimal number of light sources in the field of view. We need something like a variable length chromosome. Marques came up with the solution [10]: to represent the chromosome as a vector of vectors, where each element of the main vector (fixed length) is a combination of available pole heights and luminaire types. Each element of the main vector is associated with a secondary vector. This secondary vector has none, one or many elements (variable length) which any one of these elements represents the positioning coordinates where we can find that kind of pole/luminaire combination. Figure 3 presents an example of the proposed Marques chromosome representation.

$$c = \left\{ \begin{array}{l} P_1 L_1 \rightarrow (x_{11}^{(1)}, y_{11}^{(1)}), (x_{11}^{(2)}, y_{11}^{(2)}) \\ P_1 L_2 \rightarrow \emptyset \\ \vdots \\ P_1 L_m \rightarrow (x_{1m}^{(1)}, y_{1m}^{(1)}), (x_{1m}^{(2)}, y_{1m}^{(2)}), (x_{1m}^{(3)}, y_{1m}^{(3)}) \\ P_2 L_1 \rightarrow \emptyset \\ P_2 L_2 \rightarrow (x_{22}^{(1)}, y_{22}^{(1)}) \\ \vdots \\ P_2 L_m \rightarrow (x_{2m}^{(1)}, y_{2m}^{(1)}), (x_{2m}^{(2)}, y_{2m}^{(2)}) \\ \vdots \\ P_n L_m \rightarrow (x_{nm}^{(1)}, y_{nm}^{(1)}) \end{array} \right.$$

Figure 3: A chromosome c represented.

2.5 Genetic operators

Crossover is the genetic operator responsible for converging the population of individuals to fitter ones. Because of the unique nature of the Marques chromosome representation, a novel crossover algorithm is also here proposed. After selecting two individuals from population as parents, simply go through the elements of each main vector and randomly choose a crossover point inside their respective secondary vectors. Offspring will combine first elements of the secondary vector of one parent (left to the crossover point) with the last elements of the secondary vector of the other parent (right to the crossover points), respecting the order of elements in the main vector. In Figure 4 there is an example of a crossover between parents, shown in Figure 4(a), generating a possible offspring, presented in Figure 4(b).

$$\text{Par} = \left\{ \begin{array}{l} p1 = \left\{ \begin{array}{l} P_1 L_1 \rightarrow (10, 8) * (25, 13) \\ P_1 L_2 \rightarrow *(22, 5) (2, 16) (2, 15) \\ P_2 L_1 \rightarrow * \\ P_2 L_2 \rightarrow (9, 17) * \end{array} \right. \\ p2 = \left\{ \begin{array}{l} P_1 L_1 \rightarrow (2, 20) (9, 10) (1, 23) * (5, 6) \\ P_1 L_2 \rightarrow (18, 5) * (21, 16) (9, 19) \\ P_2 L_1 \rightarrow (12, 2) * \\ P_2 L_2 \rightarrow *(8, 21) (7, 25) \end{array} \right. \end{array} \right. \quad \text{(a) Set **Par** of chosen parents (* are crossover points).}$$

$$\text{Off} = \left\{ \begin{array}{l} o1 = \left\{ \begin{array}{l} P_1 L_1 \rightarrow \underbrace{(10, 8)}_{p1} \underbrace{(5, 6)}_{p2} \\ P_1 L_2 \rightarrow \underbrace{(21, 16)}_{p2} \underbrace{(9, 19)}_{p2} \\ P_2 L_1 \rightarrow \emptyset \\ P_2 L_2 \rightarrow \underbrace{(9, 17)}_{p1} \underbrace{(8, 21)}_{p2} \underbrace{(7, 25)}_{p2} \end{array} \right. \\ o2 = \left\{ \begin{array}{l} P_1 L_1 \rightarrow \underbrace{(25, 13)}_{p2} \underbrace{(2, 20) (9, 10) (1, 23)}_{p1} \\ P_1 L_2 \rightarrow \underbrace{(22, 5) (2, 16) (2, 15)}_{p2} \underbrace{(18, 5)}_{p1} \\ P_2 L_1 \rightarrow \underbrace{(12, 2)}_{p1} \\ P_2 L_2 \rightarrow \emptyset \end{array} \right. \end{array} \right. \quad \text{(b) Set **Off** of possible generated offspring.}$$

Figure 4: Crossover between parents $p1$ and $p2$ generating offspring $o1$ and $o2$.

Mutation is the genetic operator responsible for diversifying individuals to explore the space of solutions. The implemented mutation can trade any of the following combinations at a time: just luminaire coordinates; just pole height; just type of lamp; luminaire coordinates and pole height; luminaire coordinates and type of lamp; pole height and type of lamp; or luminaire coordinates, pole height and type of lamp.

Elitism operator is also used to grant the better individuals are kept from the last generation to the current one. It is usually limited in the range of 1 individual to 5% of the population.

2.6 Fitness function

One of the considered criteria for a green lighting design is to uniformly distribute the illuminance across the area to be illuminated. Therefore, the solution must have regions within an acceptable range of illuminance. No under-illumination is desired, due to standards and discomfort issues, and no over-illumination is allowed, due to undesirable energy wastage. For public spaces lighting design, the optimal target should be around 30 lux (illuminance category set "A").

The fitness function should reflect better fitness to individuals that have illuminance uniformly distributed around range of 30 lux. To achieve this, a two-step fitness function is here proposed. First, the evaluation of the illuminance for all points of the discretized area is performed by Equations (1) and (2). Then, the total illuminance of each point is evaluated by a normal distribution function, as shown in Figure 5. Equation (4) is the function to evaluate the partial fitness for the illuminance of a point using the normal probability density function with mean 30 lux and standard deviation of 10 lux.

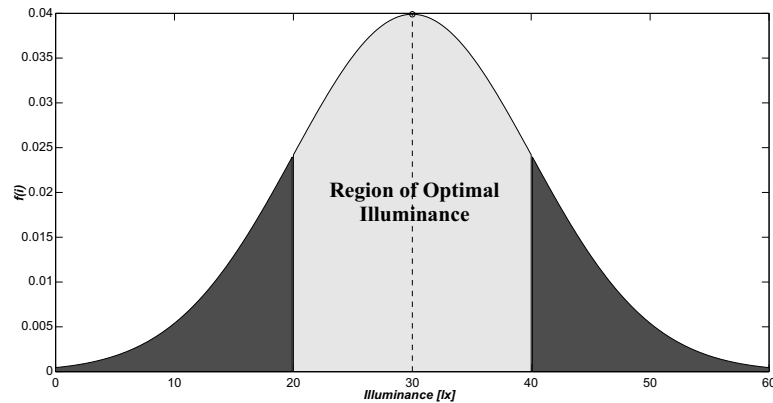


Figure 5: Partial fitness in function of calculated illuminance for a given observed point.

$$f(i) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp \left[-\frac{(E_i - \mu)^2}{2\sigma^2} \right] \quad (4)$$

where $f(i)$ is the partial fitness of a single point in the discretized area; E_i is the illuminance calculated at the observed point i , as in Equation (1); μ is the the optimal illuminance target (in this case, 30 lux); and σ is the standard deviation of the normal distribution (in this case, it is forced to be equal to the desired tolerance of 10 lux).

After, all results are summed, as presented by Equation (5). This is the total fitness for an individual from the population. Note that when there are more points of the discretized area with illuminance level inside the range of 20 to 40 lux, the greater is the fitness.

$$F(x) = \sum_{i=0}^{N-1} f(i) \quad (5)$$

where F_T is the total fitness for the individual x from the current population; N is the total number of discretized area points; and $f(i)$ is the fitness of a single point in the area.

2.7 Choosing the solution

Another criteria adopted for green lighting design is that the desired solution needs to have low power consumption. To reach this goal, a different strategy is presented. Instead of taking as the global solution the fittest individual from the last generation, the final solution will be taken from the fitter individuals separated by the elitism operator. The chosen solution is the individual from elitism which has the lower power consumption.

3. EXPERIMENTAL RESULTS

A case study was applied to a real lighting design of a parking lot in the airport of Uberlândia. Its infrastructure design is presented in Figure 6. The edified design, i.e., the initial proposed engineering solution, was conceived by a traditional process based on the engineer experience. The adopted solution is based in 4 poles with 3 fixtures, and 2 poles with 2 fixtures. All luminaires are composed with SRP222 1xSON-TPP250W lamp from Philips manufacturer.

Figure 7 shows the illuminance contour curves, also known as isolux curves, generated by Calculux software [4] for the current lighting design edified in the parking lot.

The polygons representing the area to be illuminate (the parking lot) was loaded by the implemented GA, as well as the polygons which represent forbidden regions for positioning the lighting poles (streets inside the parking lot). We run the GA

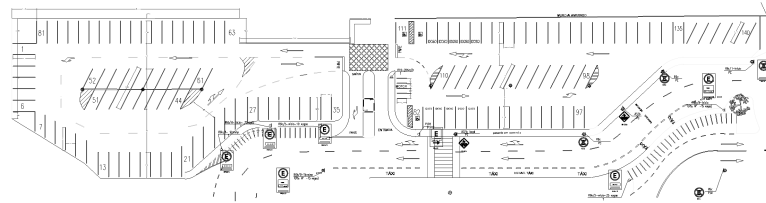


Figure 6: Infrastructure design of the parking lot at the airport of Uberlândia (UDI), Brazil.

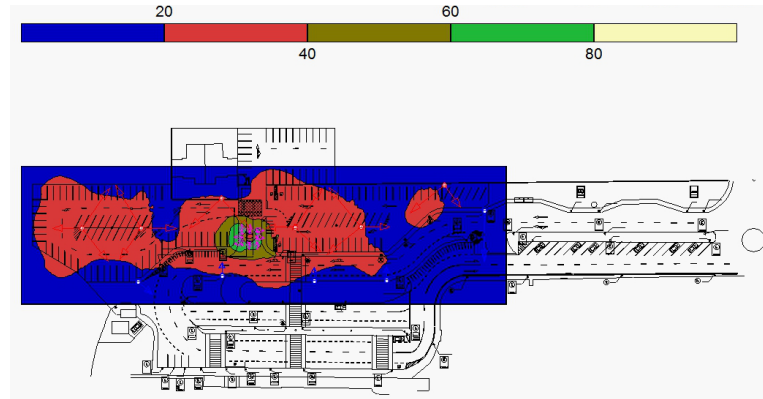


Figure 7: Isolux curves for the current lighting design edified in the parking lot, with power consumption of 4000w.

in a cluster of 8 Intel®Xeon®E5504 servers (each one with 8 cores @2.00 GHz, 48 cores total), FSB @1333 MHz, 4Gb RAM Memory, and O.S. Linux Ubuntu 10.04LTS (kernel 2.6.32). Figure 8 presents solutions achieved with 10, 100, and 1000 generations.

The final solution is achieved with 18 light poles with mixed types of lamp, as presented in Figure 8(c). The solution is better described in Table 2.

Table 2: Legend for the fittest individual from generation 1000 (Figure 8(c)).

Id	Lamp	Rated Wattage	Pole height [m]	Coordinate [m]	
				X	Y
A	HQI-T 150W/NDL	150	12	113.5	38.5
B				41.5	11.5
C				47.5	14.0
D				136.0	20.0
E	HQI-T 400W/D HQI-VG	360	12	8.0	17.0
F				26.0	35.5
G				84.0	28.5
H	CDO-TT 250W/828	244	12	54.5	36.0
I				98.0	39.5
J				111.0	16.0
K				153.0	36.5
L	CDO-TT Plus 100W/828	98	12	5.5	30.5
M	CPO-TW 60W/728	60	12	38.5	38.0
N	CPO-TW 140W/728	140	12	26.0	10.5
O				131.0	35.0
P	CPO-TW 90W/728	90	7	166.5	38.0
Q	SON 70W I	70	10	97.5	16.0
R	SON 150W/220	147	12	64.0	22.0

4. CONCLUSION

One can compare both solutions, the edified one designed by an engineer and the evolved one designed by the GA, by analyzing Figures 7 and 8(c). It is simple to observe that GA's solution is better when considering the uniformity of illuminance distribution. Another factor to be considered is the power consumption of both projects. GA's solution has an economy of almost 10% on power consumption when compared to the edified solution. Both considered criteria for green lighting design are

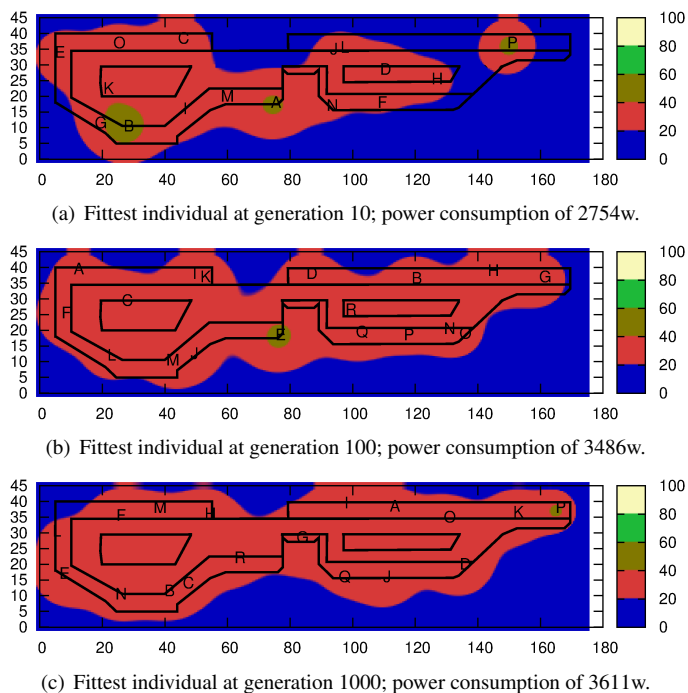


Figure 8: Isolux curves for achieved solutions by several generations in a GA run.

achieved. The implemented GA allows rational use of electric energy by avoiding energy wastes that usually are not so clear for the lighting designer.

Regarding project harmony, GA's solution lacks of it. The solution presents three different pole heights and nine different types of lamp. The geometry of poles positioning are also less harmonic than the edified solution. This is the moment when a specialist is needed. GA's artificial intelligence still does not fulfill every design criteria. We propose it to handle the infinity of possible combinations of a project in order to explore efficiency of the design.

4.1 Ongoing work

Other criteria for green lighting design have been in study to be incorporated by the presented GA application, as the minimization of illuminance outside the area of interest. Economical criteria, like edification and maintenance costs, are also objects of study to adjust the performance of GA to better fit engineering designer needs.

Another aspect of ongoing work is to exchange the hypothetical and simplified versions of fixtures and poles for the real commercial version of them.

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