# REAL-TIME CONTROL BASED ON HIGH ORDER NEURAL NETWORKS USING STOCHASTIC ESTIMATION

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**Abstract –** An adaptive discrete-time regulator system for a Furuta pendulum is presented. A high order neural network in discrete-time is used to identify the plant behavior; this network is trained with an extended Kalman filter where the associated state and measurement noises discrete-time covariance matrices are calculated with stochastic estimation. Then, the discrete-time block control and sliding mode techniques are used to develop the regulation for the angular position of a Furuta pendulum. Real-time results presented in this paper shows that the proposed method provides accurate estimation for the covariance matrices associated in the extended Kalman filter.

**Keywords –** High order neural networks, extended Kalman filter, stochastic estimation.

## 1 Introduction

The Furuta pendulum is underactuated and extremely nonlinear due to the gravitational forces and the coupling arising from the Coriolis and centripetal force. This kind of system has been widely used to demonstrate the effectiveness of different control algorithms. for example: fuzzy controller combined with proportional controller [3], backstepping control [5], linear quadratic optimization technique [8], using sliding modes [12], controlled Lagrangians [13], energy-shaping control [14], using $\mathbf{H}_\infty$ [22], among others.

On the other hand, artificial neural networks (NN) have become an attractive tool which can be used to construct a model of complex nonlinear processes ( [4], [9], [18]); particularly, the used of discrete-time high order neural networks for identification and control has recently increased ( [2], [4]) where the extended Kalman filter (EKF) based algorithms have been introduced to train these NN and to improve the learning convergence ( [1], [7], [19], [20]), among others.

In recent adaptive and robust control publications, numerous approaches have been proposed for the design of nonlinear control systems. Among those, block control (BC) combined with sliding mode (SM) constitutes a well suited design methodology ( [1], [11]). Nevertheless, as well as several feedback linearization schemes the BC technique requires non-singularity of some parameters in order to preserve controllability of NN. However, it is known that the sliding mode controllers ensures robustness of the closed-loop systems with respect to match perturbations. In practice, many control plants dynamics have both, matched and unmatched perturbations due to many parameter variations and external disturbances [21].

In this paper, we propose a discrete-time recurrent high order neural network (RHONN) trained with the EKF algorithm in order to identify the Furuta pendulum. The associated state and measurement noises discrete-time covariance matrices are calculated with a recursive algorithm based on statistical methods for random process. The proposed neural identifier is in the nonlinear block controllable (NBC) form, which allows applying the block feedback linearization control and sliding mode techniques. The block control technique is used to design a nonlinear sliding manifold such that the resulting sliding mode dynamics are described by a desired linear system. The combine approach, which includes the identification of the plant with the neural network and the application of the block control and sliding modes control techniques, ensures robustness with respect to external disturbances and stable desired sliding motion over the designed proposed sliding manifold. The proposed neural identifier and block sliding mode control applicability are illustrated by the swing up the Furuta pendulum and the regulation of the angular position via real-time results.

## 2 Discrete-time dynamical model based on recurrent high order neural networks

In order to introduce the proposed method, in this paper we consider that the plant to be controlled has the nonlinear discrete-time structure represented in state space as:

$$\chi(k+1) = F(\chi(k), u(k), k) + q(k) \quad (1)$$
$$y(k) = h(\chi(k)) + v(k) \quad (2)$$

where $u(k)$ is the input to the system; $q(k)$ and $v(k)$ are independent noises of process and measurement, respectively; $F(\bullet)$ is the nonlinear transition matricial function, possibly time-varying; $h(\bullet)$ is the nonlinear measurement function, possibly time-varying too. It is important to mention that in this paper we consider the discrete-time Furuta pendulum model with structure (1-2) and its parameters are unknown. Nevertheless we have available the full state measurements which consist of: $\chi_1(k) = \alpha(k)$ that represents the pendulum angle, $\chi_2(k) = \theta(k)$ is the arm position angle, $\chi_3(k) = \omega_\alpha(k)$ and $\chi_4(k) = \omega_\theta(k)$ are the respective angular speed for $\alpha(k)$ and $\theta(k)$. Model (1-2) is presented to show the structure of the Furuta pendulum model, and this structure will be used for the neural network identifier design in the following sections.

### 2.1 Discrete-time Recurrent High Order Neural Networks

A high order neural network is a NN which not only a linear combination of the components, but also of their products are considered, as it is explained in [9]. The RHONN model is flexible and allows incorporating to the neural model a priory information about the system structure with less units. Additionally with this kind of NN it is possible to reduce the identification error by increasing the number of adjustable weights as well as the high order terms [17]. In this paper, we consider the problem to approximate the general discrete-time nonlinear system (1-2) by the following discrete-time RHONN series-parallel representation [17]

$$x_i(k+1) = w_i^T z_i\left(\chi(k), u(k)\right), \; i = 1, \cdots, n \tag{3}$$

where $x_i$, $i = 1, \cdots, n$ is the state of the $i$th neuron, $\chi(k)$ denotes the plant state, $n$ is the state dimension, $w_i$, $i = 1, \cdots, n$ is the respective on-line adapted weight vector, and $z_i\left(\chi(k), u(k)\right)$ is given as in [1]. This RHONN is trained with the EKF algorithm.

On the other hand, it is known that Kalman filtering estimates the state of a linear system with additive state and output noises ( [6], [20]). Due to the fact that the neural network mapping is nonlinear, an EKF-type is required [18]. The training goal is to find the optimal weight values which minimize the prediction error. In this paper, we use the EKF described as in [1].

### 2.2 Stochastic solution for the estimation of noise

The proposed stochastic formulation requires the efficient compute of $Q_i(k)$ and $R_i(k)$ in recursive form such that minimizes the identification error [1]

$$min_{Q_i(k)}\left(\chi(k) - x(k)\right) \tag{4}$$
$$min_{R_i(k)}\left(y(k) - \bar{y}(k)\right) \tag{5}$$

This can be done such that minimizes the variance ($\sigma$)

$$\sigma(\chi) = E\left(\left[\chi(k) - \chi_m(k)\right]^2\right) \tag{6}$$
$$\sigma(y) = E\left(\left[y(k) - y_m(k)\right]^2\right) \tag{7}$$

where $\chi_m(k) = E\left(\chi(k)\right)$ and $y_m(k) = E\left(y(k)\right)$ are expressed in terms of the recursive expectation value, $E(\bullet)$, that represent the instantaneous mean value of the signal. $Q_i(k)$ and $R_i(k)$ defined as in [1] are computed as

$$Q_i(k) = I\sigma(\chi) \tag{8}$$
$$R_i(k) = \sigma(y) \tag{9}$$

where $I \in \Re^{L_i \times L_i}$ is the identity matrix. Assume for this end that the cumulative mean value of $\chi(k)$ and $y(k)$ in the time interval $1 \leq j \leq k$ is approximated by [15],

$$\chi_m(k) = \frac{1}{k+1}\sum_{j=1}^{k}\chi(j) \cong E\left(\chi(k)\right) \tag{10}$$

$$y_m(k) = \frac{1}{k+1}\sum_{j=1}^{k}y(j) \cong E\left(y(k)\right) \tag{11}$$

where $\chi_m(k)$ and $y_m(k)$ are the estimation of the expectation value. In recursive form for each time instant $k$, (10) and (11) can be redefined through the recursive relation

$$\chi_m(k) = \frac{k}{k+1}\chi_m(k-1) + \frac{1}{k+1}\chi(k) \tag{12}$$
$$y_m(k) = \frac{k}{k+1}y_m(k-1) + \frac{1}{k+1}y(k) \tag{13}$$

The relation above has been used as the basis for a method of Kalman filter tuning using real-time information. A sliding windows-based approach has been combined to resolve localized information. In this approach, a sliding window frame of fixed size, say $\tau$, is shifted regularly throughout the data span from the beginning of the record to the end of the data.

Similarly to (12-13), the cumulative mean value for a window size $\tau$, can be computed through the recursive relation

$$\chi_m(k) = \frac{\tau}{\tau+1}\chi_m(k-1) + \frac{1}{\tau+1}\chi(k) \quad (14)$$

$$y_m(k) = \frac{\tau}{\tau+1}y_m(k-1) + \frac{1}{\tau+1}y(k) \quad (15)$$

It should be stressed that $\tau$ can range from a single sample to the entire length of the record. In the analysis of highly nonstationary signals, large values of $\tau$ may obscure the analysis of temporal changes occurring at segments of the observed record. The selection of an optimal window size is an important but difficult problem and will be addressed in future research.

# 3 Neural Identification and Control

In this section, we consider the problem to identify the Furuta pendulum with structure (1-2) using a RHONN with general structure (3) trained with the EKF algorithm described as in [1]

## 3.1 Furuta Pendulum Identification

We propose the neural identifier (3) in the following NBC form

$$x_1(k+1) = w_{11}(k)S(\alpha(k)) + \bar{w}_1\omega_\alpha(k) \quad (16)$$

$$x_2(k+1) = w_{21}(k)S(\theta(k)) + \bar{w}_2\omega_\theta(k) \quad (17)$$

$$x_3(k+1) = w_{31}(k)S(\alpha(k))\,S(\omega_\theta(k)) + w_{32}(k)S(\omega_\theta(k)) + \bar{w}_3u(k) \quad (18)$$

$$x_4(k+1) = w_{41}(k)S(\alpha(k))\,S(\omega_\theta(k)) + w_{42}(k)S(\omega_\theta(k)) + \bar{w}_4u(k) \quad (19)$$

where $x_i(k), i = 1, 2, 3, 4$ is the state of the $i$th neuron; $x_1(k)$ is the estimate of the angular position $\alpha(k)$, $x_2(k)$ is the estimate of the angular position $\theta(k)$, $x_3(k)$ is the estimate of the angular speed $\omega_\alpha(k)$, $x_4(k)$ is the estimate of the angular speed $\omega_\theta(k)$, $u(k)$ is the input voltage to the motor, $w_{11}(k)$, $w_{21}(k)$, $w_{31}(k)$, $w_{32}(k)$, $w_{41}(k)$, $w_{42}(k)$ are the respective on-line adapted weights. The weights $\bar{w}_1$, $\bar{w}_2$, $\bar{w}_3$ and $\bar{w}_4$ are constant positive values, which are selected initially arbitrarily and modified in order to reduce the identification error. Finally $S(\bullet)$ is defined as in [2]. We use a neural model due to the fact that the parameters and external disturbances of the plant to be controlled are unknown; then, to approximate the plant complete model, we propose a neural identifier which reproduces the behavior of the plant, without the identification of the plant parameters. Note that the proposed NN identifier (16-19) belong to the triangular structure. However, this NN identifier has a strict feedback form [10] (or block controllable form [11]). The advantage of the identifier is that it allows the application of the block control and sliding modes technique [2]. Note that the NBC form is a particular case of the triangular form where each $i$th block is linear with respect to the vector $\chi^{i+1}$. So, the control algorithm will be designed based on the proposed NN identifier (16-19). Moreover, the structure (16-19) uses the smallest quantity of weights needed to obtain an adequate identification and reduces computational requirements. It is used the EKF training algorithm and is performed on-line using a series-parallel configuration [17], due to the fact that this configuration constitutes a well approximation method of the real plant by the neural identifier and improve the learning convergence. In this paper, the associated state $Q_i(k)$ and measurement $R_i(k)$ noises discrete-time covariance matrices are computed as (8-9) as it is explained in subsection 2.2. All the NN states are initialized in a random way as well as the weights vectors. It is important to note that the initial conditions of the NN are completely different from the initial conditions for the plant. The control scheme is based on the discrete-time block control and sliding modes procedure explained in [2]. This procedure is applied to the neural identifier (16-19).

## 3.2 Block controller design

Given full state measurements, the control objective is to develop the regulation of the angular position $\alpha(k)$ for the Furuta pendulum with structure (1-2), using the discrete-time block control and sliding modes techniques. Defining $\chi^1(k) = \begin{bmatrix} \alpha(k) \\ \theta(k) \end{bmatrix}$, $\chi^2(k) = \begin{bmatrix} \omega_\alpha(k) \\ \omega_\theta(k) \end{bmatrix}$, $z^1(k) = \begin{bmatrix} S(\alpha(k)) & 0 \\ 0 & S(\theta(k)) \end{bmatrix}$, $z^2(k) = \begin{bmatrix} S(\alpha(k))\,S(\omega_\theta(k)) \\ S(\omega_\theta(k)) \end{bmatrix}$, $w^1(k) = \begin{bmatrix} w_{11}(k) & 0 \\ 0 & w_{21}(k) \end{bmatrix}$, $w^2(k) = \begin{bmatrix} w_{31}(k) & w_{32}(k) \\ w_{41}(k) & w_{42}(k) \end{bmatrix}$, $\bar{w}^1(k) = \begin{bmatrix} \bar{w}_1(k) & 0 \\ 0 & \bar{w}_2(k) \end{bmatrix}$ and $\bar{w}^2(k) = \begin{bmatrix} \bar{w}_3(k) & 0 \\ 0 & \bar{w}_4(k) \end{bmatrix}$. Then system (16-19) can be represented as the block control form consisting of two blocks:

$$\left\{x^1(k+1) = w^1(k)z^1\left(\chi^1(k)\right) + \bar{w}^1\chi^2(k)\right. \quad (20)$$

$$\left\{x^2(k+1) = w^2(k)z^2\left(\chi^2(k)\right) + \bar{w}^2u(k)\right. \quad (21)$$

where $x^1 = \begin{bmatrix} x_1 & x_2 \end{bmatrix}^T$, $x^2 = \begin{bmatrix} x_3 & x_4 \end{bmatrix}^T$, $x^i$, $i = 1, 2$ is the neuron state of the $j$th block, $n$ is the block dimension, $j$ is the number of blocks, $j = 1, 2$, $n_1 = n_2 = 2$.

Defining $\chi^{1d}(k)$ as the angular position reference and to control the angular position $\alpha(k)$, we define the error $\varepsilon^1(k) = \chi^1(k) - \chi^{1d}(k)$ and applying the block control procedure (as explained in [2]) the system (16)-(19) in the new variables $\varepsilon^1(k)$ - $\varepsilon^2(k)$ is presented of the form

$$\left\{\varepsilon^1(k+1) = \mathbf{k}_1\varepsilon^1(k) + \bar{w}^1\varepsilon^2(k) + \tilde{\Delta}^1(k)\right. \quad (22)$$

$$\left\{\varepsilon^2(k+1) = f(\chi(k),k) + \bar{w}^2 u(k) + \tilde{\Delta}^2(k)\right. \quad (23)$$

where $f(\chi(k),k) = \begin{bmatrix} \varsigma_1 & 0 \\ 0 & \varsigma_2 \end{bmatrix}$ with $\varsigma_1 = w_3(k)z_3(\chi(k)) - \chi_{3d}(k)$, $\varsigma_2 = w_4(k)z_4(\chi(k)) - \chi_{4d}(k)$, $\tilde{\Delta}^1(k) = \begin{bmatrix} \tilde{\Delta}_1(k) & 0 \\ 0 & \tilde{\Delta}_2(k) \end{bmatrix}$ and $\tilde{\Delta}^2(k) = \begin{bmatrix} \tilde{\Delta}_3(k) & 0 \\ 0 & \tilde{\Delta}_4(k) \end{bmatrix}$ with $\Delta_i(k+1)$, $i = 1,2,3,4$, and $\|\mathbf{k}\| < 1$ (for more explanation about getting $\mathbf{k}$ see [2]). Selecting the sliding variable as $s(k) = \varepsilon^2(k)$ and taking into account saturation values $\|u(k)\| \leq u_0(k)$ with $u_0(k) > 0$ of the voltage applied to the motor, the following control is implemented [21]:

$$u(k) = \begin{cases} \tilde{u}_{eq}(k) & \text{for } \|\tilde{u}_{eq}(k)\| \leq u_0(k) \\ u_0(k)\frac{u_s(k)}{\|u_s(k)\|} & \text{for } \|\tilde{u}_{eq}(k)\| > u_0(k) \end{cases} \quad (24)$$

with

$$\begin{aligned} \tilde{u}_{eq}(k) &= -\left[\bar{w}^2\right]^{-1}\left[s(k) + f(\chi(k),k)\right] + \left[\bar{w}^2\right]^{-1}\left[f(\chi(k-1),k-1)\right] + \tilde{u}_{eq}(k-1) \\ u_s(k) &= -\left[\bar{w}^2\right]^{-1} f(\chi(k),k)\,. \end{aligned}$$

# 4 Real-time results

This section presents the real-time results corresponding to identification and control of a Furuta pendulum with structure (1-2). The experiments are performed using a benchmark, which includes:

- A Mechatronics Control Kit from Quanser Consulting Inc.
- The software Code Composer Studio supplied with the DSK Board, the TI C6x Optimizing C-compiler, the Code Composer Development/Debug IDE (integrated development environment), as well as DSP BIOS/RTDX real-time debugging/plotting capabilities.
- A Furuta pendulum.
- A Texas Instruments DSP development system.
- The TMS320C6713 DSK Board (a DSP board with USB interface).
- PWM/Optical Encoder data Acquisition Daughter Board.
- PWM amplifier.
- 1000 counts/rev optical encoder.
- An interface board for running Quanser experiments.
- A PC for supervision.

Fig. 1 displays a photograph of the complete prototype for the Furuta pendulum. For more details about the Mechatronics Control Kit consult [16].

## 4.1 Identification

For the identification process, we consider the complete plant state measurement; we also consider that noise is added to the measurement process. The covariance matrices $Q_i(k)$ and $R_i(k)$ are initialized as diagonals with random values. These initial values are adjusted recursively in order to minimize the identification error, as is explained in section 2.2 using a window size of 2 samples. The identification results are included as follows: in Fig. 2 we show the identification process for angular position $\alpha(k)$ (by simplicity and due to limit of space, we only present the estimation of this state). In this Fig. we present three signals which correspond to: the output signal for the real plant (dashed line), the identification result with the proposed stochastic estimation method (bold line), and the identification with the existent method (thin line). As it can be seen in Fig. 2, the proposed stochastic estimation method provides better accuracy than the existent method.

Real-time results presented in Figs. 2 show that the proposed method provides accurate estimation of non-stationary effects. This information may be important in determining strategies for control and special protection systems. It is important to note that the proposed real-time identification method presents as good time convergence as the ones presented in [1].

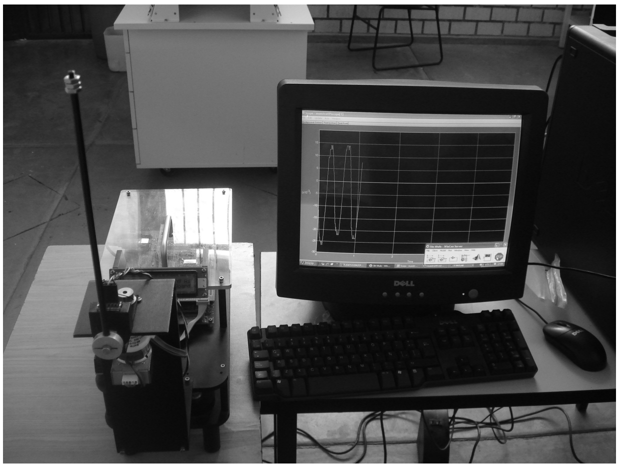

Figure 1: Photograph of the complete prototype for the Furuta pendulum.

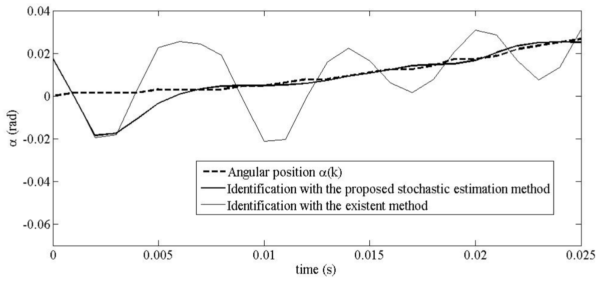


Figure 2: Angular position identification: $x_1(k)$ that identifies $\chi_1(k) = \alpha(k)$.

### 4.2 Swing-up and regulation

The swing-up performance for the Furuta pendulum as well as the regulation results for the plant output $\alpha(k)$ are verified in Fig. 3. In this figure $\alpha_r(k)$ represents the reference angular position and $\alpha(k)$ the position output. We introduce disturbances at different times in order to asses the pendulum angular position recovery. These disturbances are created by moving the mass of the pendulum. The parameters of the neural controller are: $u_0 = 10\ Volts$ which represents the voltage applied to the motor, $\mathbf{k}_1 = \mathbf{k}_2 = \mathbf{k}_3 = \mathbf{k}_4 = -0.95$; $\bar{w}_1 = \bar{w}_2 = 0.0015$, $\bar{w}_3 = \bar{w}_4 = 0.0001$; the real time experiments are performed with a sampling time of $1ms$.

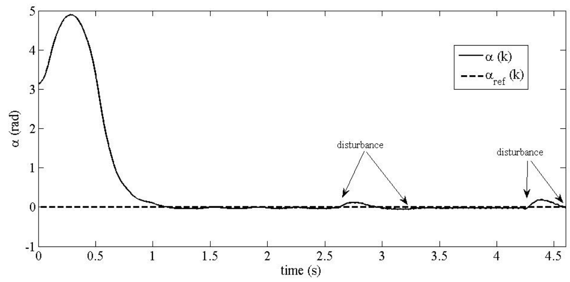


Figure 3: Swing-up and angular position regulation performance: $\alpha_{ref}(k)$ (dashed line), $\alpha(k)$ (solid line).

According to Figs. 2-3, can be seen that statistical techniques are needed to help identify relevant dynamics from noise or random effects in measured data. It is important to remark that this control scheme does not require the knowledge of the plant parameters neither the external disturbance.

## 5 Conclusions

This paper has presented the real-time identification and control scheme for a Furuta pendulum using a recurrent high order neural network. Based on the discrete-time block control technique is possible to develop angular position regulation for this plant. The training of the neural network is performed on-line using an extended Kalman filter in a series-parallel configuration where the associated state and measurement noises discrete-time covariance matrices are computed using stochastic solution for the estimation of noise. Real-time results present robustness of the proposed control methodology with respect to external disturbances and these results validate the applicability of the proposed approach.

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