NON-NEGATIVE MATRIX FACTORIZATION FOR IMPROVING PASSIVE SONAR SIGNAL DETECTION

N. N. de Moura, Igor Paladino and J. M. de Seixas
Signal Processing Laboratory
{natmoura, igor.costa, seixas}@lps.ufrj.br

Abstract – Non-negative matrix factorization (NMF) has been shown to be useful for decomposition of multivariate data. In this paper, NMF will be implemented using the alpha divergence and obtain blind signal separation (BSS). The aim is to improve the signal/interference ratio of a passive sonar system that suffers from mutual interference in adjacent bearings and from the self-noise.

Keywords – Passive Sonar, Spectral Analysis, DEMON Analysis, Blind Source Separation, Non-Negative Matrix Factorization, Divergence.

1. INTRODUCTION

Unsupervised learning algorithms [1,2], may be understood as a factorization of a data matrix, subject to restrictions. Depending on the restriction used, the resulting factorization can access to different properties. For example, the Principal Component Analysis (PCA) imposes an orthogonality constraint, resulting in a maximum variance representation.

In the case of NMF (Non-negative Matrix Factorization), the purpose is to decompose a data matrix in two others matrices that represents the original data matrix. As the data matrix does not have to be negative for the performance of its factorization, this restriction means that it may have already learned part of data representation [3]. A matrix of basis vectors is created and used in distribution to generate a sparse significance in the reconstruction of the data [4]. An analysis was made using the alpha divergence [3] as cost function on NMF factorization in an experimental data matrix that was acquired from a passive sonar system. Tests were implemented with the experimental data with the aim of improving signals detection (signal/interference enhancement) from a particular direction that are suffering interference from adjacent bearing and the self-noise.

Section 2 describes the proposed approach, how the signals were acquired in a determined direction, the DEMON (Detection Envelope Modulation On Noise) analysis which performs a spectral analysis with the aim of detecting and identifying signals from a particular direction. A description of the alpha divergence with different values of α as cost function on NMF implementation. Section 3 shows the results that were achieved with the application of the divergence with different α parameter in order to measure these performances and the mutual information was used to quantify the performance of each α value. Section 4 make a conclusion and future works are proposed.

2. PROPOSED APPROACH

Among various functions, a passive sonar system [5] aims to accomplish the estimation of direction of arrival (DOA - Direction Of Arrival) [6,7] of a particular contact from a determined direction. Figure 1 shows the DOA estimation of a passive sonar system. The horizontal axis represents the bearings from 0 to 180 to the right and from 0 to 180 to left. This is obtained from an array of sensors that makes the omnidirectional surveillance. The vertical axis represents the time acquisition window, which, in this case considers one second length. The energy measured in a bearing at a given time window has a gray scale representation. As greater the intensity of gray color, higher the probability of a contact to be in that direction. This allows the sonar operator to use this display to do a first investigate for contact detection in a determined direction. After DOA estimation, a contact identification is required, this is accomplished by DEMON analysis.

2.1 DEMON Analysis

DEMON analysis [8] is a narrow band analysis that operates on cavitation noise to provide the contact propulsion [9]. This analysis aims at estimating the number of shafts and number of blades of the contact, which quite useful for contact identification.

Figure 2 shows the typical block diagram for DEMON analysis.

Given a particular direction of interest, the signal is band limited by a bandpass filter (between 1 and 2 kHz, in this work). According to [7], this cavitation band provides adequate information for contact (the target) identification. The signal is demodulated [10,11] and it is resampled [12] in order to match the frequency range of the propulsion information (shaft rotation and blade rate). In our case, the sampling rate is of $fs = 31,250$ Hz, thus, it was necessary to implement two resampling by a factor of 25 in cascade to obtain a frequency range of 0 and 25 Hz which corresponds to 0 to $1,500$ rpm (rotation per minute). This range for which the relevant information for identifying the contact propulsion stays. In sequence, a fast Fourier transform algorithm [13] is applied and TPSW (Two Pass Split Window) algorithm [8] is implemented to remove the background noise and equalize the frequency peaks allowing to emphasize the smaller peaks.

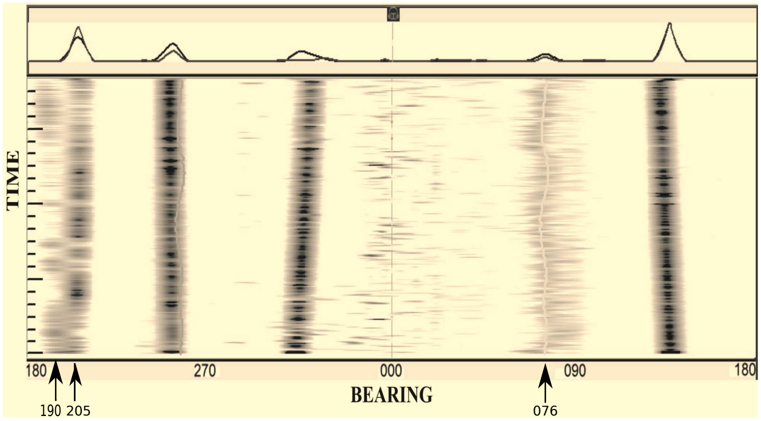


Figura 1: Bearing×Time display of a passive sonar system.

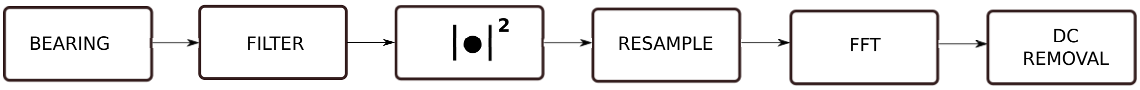


Figura 2: Block diagram of a DEMON analysis.

Figure 3 shows the resulting DEMON analysis for different bearing positions. One can depict from this figure that there is interferences between the bearing position (190^o and 205^o) and the self-noise (076^o) produces in both bearings.

2.2 Non-negative matrix factorization

The purpose of the NMF is, given a non-negative matrix $\mathbf{V}$ find two matrices $\mathbf{W}$ and $\mathbf{H}$ such that:

$$\mathbf{V} \approx \mathbf{WH} \tag{1}$$

where $\mathbf{W}$ and $\mathbf{H}$ must be greater than 0. For the passive sonar signal problem the $\mathbf{V}_{\mathbf{n}\times\mathbf{m}}$ is a matrix where each column contain the frequency bins which represents the mixed data matrix. The matrix $\mathbf{W}_{\mathbf{n}\times\mathbf{r}}$ can be seen as a basis vector that is optimized by linear approximation of matrix $\mathbf{V}$ and the matrix $\mathbf{H}_{\mathbf{r}\times\mathbf{m}}$ would be the matrix of independents sources. Another way to express the equation 1 is column by column as $\mathbf{v} \approx \mathbf{Wh}$, where $\mathbf{v}$ and $\mathbf{h}$ are the corresponding columns of $\mathbf{V}$ and $\mathbf{H}$ respectively. Thus, each vector data $\mathbf{v}$ is approximated by a linear combination of the columns of $\mathbf{W}$, which is weighted by the components $\mathbf{h}$. Then, the central problem is to estimate the matrix $\mathbf{H}$ that contains the independent components. The objective of this paper is to use the alpha divergence as cost function to implement the NMF.

2.3 Divergence

The measure of divergence and the (dis)similarity are important in many fields of signal processing such as estimation, interference enhancement, optimization and learning. In general mode, it measures a difference between two n-dimensional probability distributions[1] [3]. The divergence between two distributions can be expressed mathematically by the equation:

$$D(\mathbf{V}||\mathbf{Q}) = \sum_{i=1}^{n} d(v_i, q_i). \tag{2}$$

Where $\mathbf{Q} = \mathbf{WH}$. The objective is to minimize $D(\mathbf{V}||\mathbf{Q})$ to search for the NMF factorization. In this work was used the alpha divergence as cost function to minimize the error to search de equation 1.

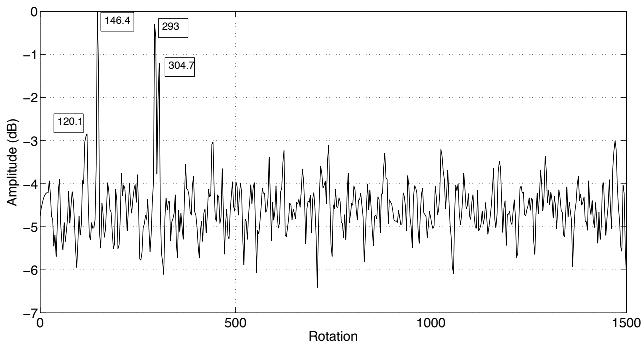
2.3.1 Alpha Divergence

The Alpha divergence will be used to implement a non-negative matrix factorization and find blindly sources (in frequency-domain) from a DEMON analysis. It can be computed from:

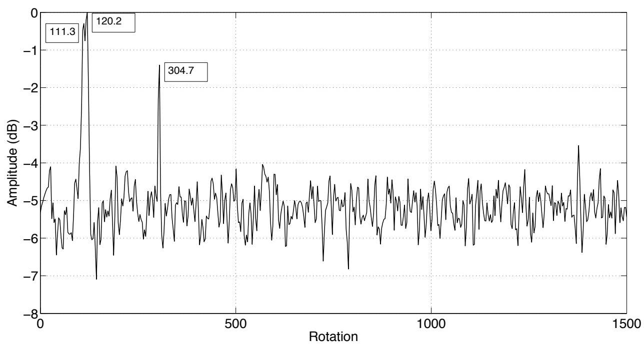
$$D_A^{(\alpha)}(\mathbf{v}||\mathbf{q}) = \frac{1}{\alpha(\alpha-1)} \sum_{ik} (v_{ik}^{\alpha} q_{ik}^{1-\alpha} - \alpha v_{ik} + (\alpha - 1) q_{ik}), \qquad \alpha \in \mathbb{R}. \tag{3}$$

For the special cases, when $\alpha = 2, 0.5, -1$, these are respectively, the Chi-square, the Hellinger and Pearson [3]. The equation of each derivatives of alpha divergence is shown below.

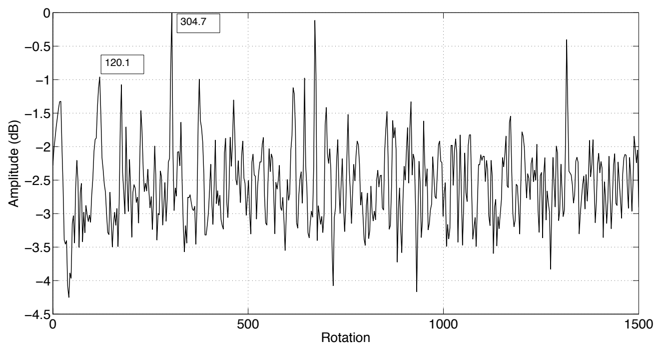
[1]Matrices will be used to represent the distributions of probability.



(a) Bearing 190.



(b) Bearing 205



(c) Bearing 076

Figura 3: DEMON analysis at bearings: (a) 190^o, (b) 205^o and (c) 076^o.

$$D_A^{(2)}(\mathbf{v}||\mathbf{q}) = D_P(\mathbf{v}||\mathbf{q}) = \frac{1}{2}\sum_i \frac{(v_i - q_i)^2}{q_i}; \tag{4}$$

$$D_A^{(1/2)}(\mathbf{v}||\mathbf{q}) = 2D_H(\mathbf{v}||\mathbf{q}) = 2\sum_i (\sqrt{v_i} - \sqrt{q_i})^2; \tag{5}$$

$$D_A^{(-1)}(\mathbf{v}||\mathbf{q}) = D_N(\mathbf{v}||\mathbf{q}) = \frac{1}{2}\sum_i \frac{(v_i - q_i)^2}{q_i}. \tag{6}$$

2.4 UPDATE OF THE MATRICES W AND H

In this work, alpha divergence will be used as a cost function to update the matrices $\mathbf{W}$ and $\mathbf{H}$. At each iteration, new estimates for $\mathbf{W}$ and $\mathbf{H}$ are found by multiplying the current value by some factor that depends on the quality of the approximation of the equation 1. May be proved that the quality of the approximation improves monotonically with the application of these multiplicative rules [14]. Repeated iterations of the update rules is guaranteed to converge to a locally optimal matrix factorization. The gradient of the equation 3 can be expressed in function of $\mathbf{W}$ and $\mathbf{H}$ as:

$$\frac{\partial D_A^\alpha}{\partial w_{ij}} = \frac{1}{\alpha}\sum_{k=1}^{N} h_{jk}\left[1-\left(\frac{v_{ik}}{q_{ik}}\right)^\alpha\right] \quad (7)$$

$$\frac{\partial D_A^\alpha}{\partial h_{jk}} = \frac{1}{\alpha}\sum_{i=1}^{m} w_{ij}\left[1-\left(\frac{v_{ik}}{q_{ik}}\right)^\alpha\right] \quad (8)$$

Instead of applying the standard gradient descent as shown early, may be used the projected gradient approach [14], which leads directly to a new learning algorithm.

$$w_{ij} = w_{ij}\left(\frac{\sum_{k=1}^{N}(v_{ik}/q_{ik})^\alpha h_{jk}}{\sum_{t=1}^{N} h_{jt}}\right)^{\frac{1}{\alpha}} \quad (9)$$

$$h_{jk} = h_{jk}\left(\frac{\sum_{i=1}^{m}(v_{ik}/q_{ik})^\alpha}{\sum_{p=1}^{m} w_{pj}}\right)^{\frac{1}{\alpha}} \quad (10)$$

3. RESULTS

Figure 4 shows the block diagram of how the divergence was applied to perform the blind source separation in order to improve the signal/interference between the bearings and the self-noise.

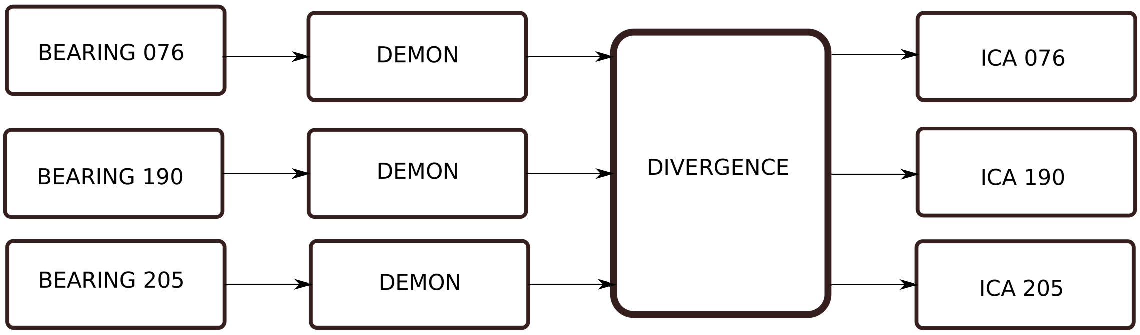


Figura 4: Block diagram of the blind source separation using the divergence.

A DEMON analysis was performed at each bearing and a data matrix was created, where each column corresponds a bearing and a non-negative matrix factorization was performed on the data matrix using the alpha divergence as cost function to extract each component.

After the component extraction it is necessary verify the similarity between the bearings and the components to certificate the quality of the extraction in quantitative mode. This is done using the mutual information [15]. The table 1 shows the mutual information between the bearings and the components.

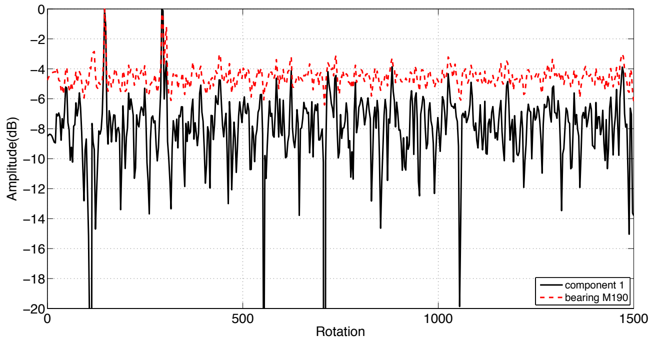
Tabela 1: Mutual information between bearings M076, M190 and M205 and the components. Pearson ($\alpha = 2$), Hellinger ($\alpha = 0.5$) e Neyman ($\alpha = -1$).

Signals Mixed	**Pearson divergence**			**Hellinger divergence**			**Neyman divergence**		
	comp1	comp2	comp3	comp1	comp2	comp3	comp1	comp2	comp3
M190	**0.7493**	0.1308	0.1693	**0.7360**	0.1385	0.1651	**0.6443**	0.2451	0.2164
M205	0.0040	**0.8189**	0.1190	0.0077	**0.8267**	0.1286	0.2108	**0.6687**	0.0687
M076	0.2062	0.1619	**0.8165**	0.2233	0.1539	**0.8221**	0.2832	0.3708	**0.8695**

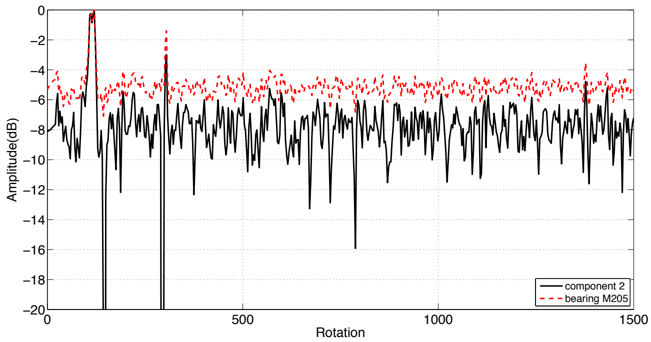
May be seen that the mutual information for each α, provides a higher value between a bearing and a component. It is possible to conclude that the component one (comp1) belongs to bearing M190, the component two (comp2) belongs to M205 and the component three (comp3) belongs to M076.

The following Figures show the components extracted using the NMF with different values of α.

A comparison between raw-data and the components at each bearing shows that the background noise suffered an attenuation in all value used of α. In all values of α the background noise suffered an attenuation about -2dB. The interferences between bearings and the self-noise were attenuate significantly. In Figure 5(a), the interference relative to bearing 205 ^{o} suffered an attenuation of -17dB and the relative to the self-noise attenuated about -2dB. Figure 5(b) the self-noise attenuated -1.5dB. The same happens to Figures 6 and 7, with different values of attenuation. This will facilitate the sonar operator on the identification of a target.

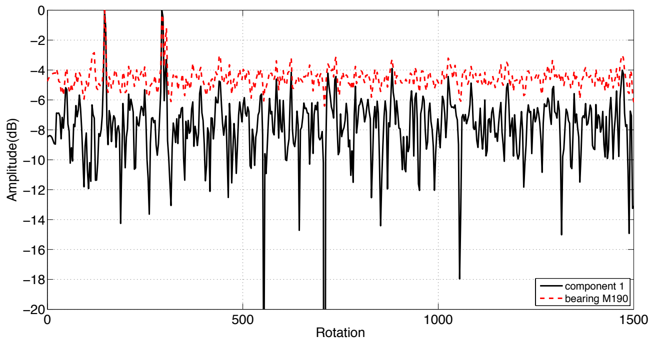


(a) Bearing 190 and Component 190.

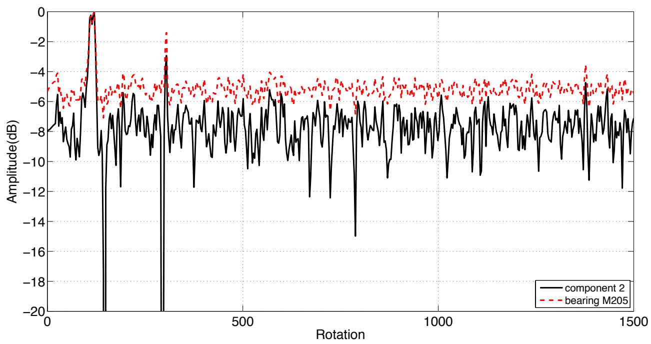


(b) Bearing 205 and Component 205

Figura 5: DEMON analysis of Pearson divergence.



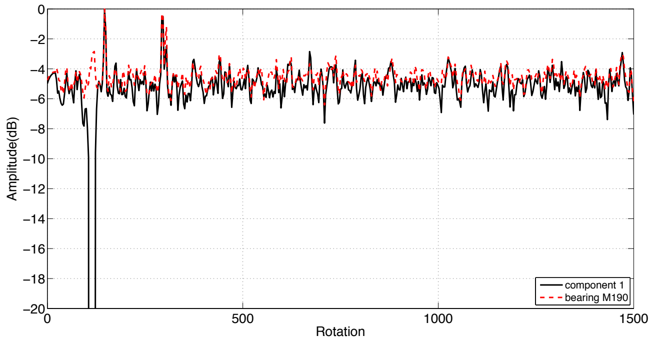
(a) Bearing 190 and Component 190.



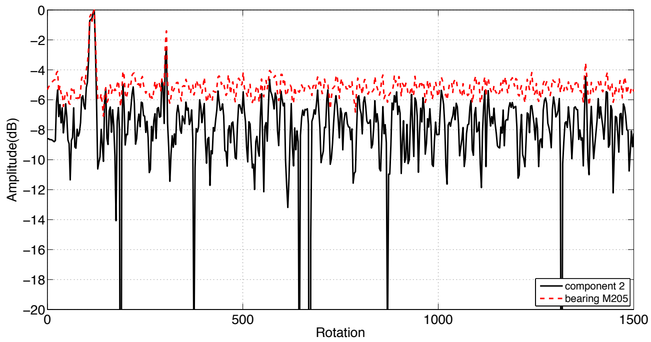
(b) Bearing 205 and Component 205

Figura 6: DEMON analysis of Hellinger divergence.

The error of each divergence is showed at the Figure 8. It was calculated using following equation:



(a) Bearing 190 and Component 190.



(b) Bearing 205 and Component 205

Figura 7: DEMON analysis of Neyman divergence.

$$error = \frac{\sum \mathbf{V}^{\alpha}(\mathbf{WH})^{1-\alpha} - \alpha\mathbf{V} + (1-\alpha)(\mathbf{WH})}{\alpha(\alpha-1)} \tag{11}$$

At each iteration the values of $\mathbf{W}$ and $\mathbf{H}$ are substituted on equation 11 to verify the convergence if the cost function.

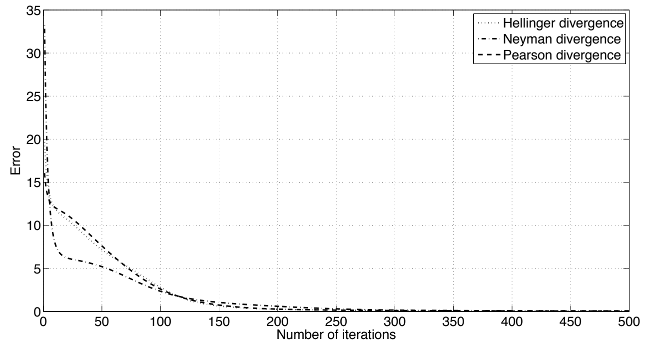


Figura 8: Errors at each divergence.

May be observed that, although the errors decrease differently to each α value for a determined iteration value, approximately at iteration 100 they remain the same.

4. CONCLUSION

The passive sonar system main purpose is to detect and identify a target from a determined direction of arrival. In some situations when the bearings are so closed enough, interferences may may arise and difficult the detection by sonar operator. A preprocessing on the raw-data must be implemented to extract the components and improve the signal/interference between the bearings and the self-noise. This is done using blind source separation to extract the components at each direction.

The non-negative matrix factorization was used to implement the blind source separation and the alpha divergence was used as cost function to obtain the optimal matrices $\mathbf{W}$ and $\mathbf{H}$ that correspond the basis vector of the subspace and the independent components to be extracted. Various values of α were applied to verify the performance of the extraction.

The mutual information was used to quantify the performance of each α value. Table 1, shows mutual information between the bearings and components. May be seen that for each value of α, a mutual information is largest enough to confirm that bearing is relative to the component. Then, it is possible to stablish which component belongs to a bearing.

A qualitative performance of the blind separation is shown in the Figures 5, 6 and 7 respectively. A great attenuation occurred between the interferences at each bearing. May be seen that the background noise suffered a significant attenuation to all α value. These attenuations will facilitate the sonar operator on detection and identification of a contact.

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