Robust Fuzzy Digital PID Control for Uncertain Dynamic Systems with Time Delay

Danúbia Soares Pires and Ginalber Luiz de Oliveira Serra
Department of Electroelectronics
Laboratory of Computational Intelligence Applied to Technology
Federal Institute of Education, Science and Technology
São Luis-MA, Brazil
Email: danubiapires@ifma.edu.br and ginalber@ifma.edu.br

Abstract—In this paper, a robust fuzzy digital PID control strategy, via multiobjective genetic algorithm, based on the gain and phase margins specifications, with applications to uncertain dynamic systems with time delay, is proposed. A mathematical formulation based on the gain and phase margins, the fuzzy model and PID digital controller structures, and the robust stability of the uncertain dinamic system with time delay, is deduced. A multiobjective genetic strategy is defined to tune the fuzzy controller parameters, so the gain and phase margins specified to the fuzzy control system are get. It is proposed of two theorems related to necessary and suficient conditions for fuzzy robust digital control design. The first theorem enunciates that the robust PID controller in the *i-th* rule of the fuzzy robust digital PID controller guarantees the gain and phase margins specifications for the corresponding linear model. The second theorem enunciates that the robust PID controller in the *i-th* rule of the fuzzy robust digital PID controller guarantees the stability for all linear models. Experimental results shown the eficiency of the proposed methodology through tracking of the reference and the gain and phase margins keeping closed of the specified ones.

Index Terms—Fuzzy Control, Digital PID Controller, Uncertain Dynamic Systems, Multiobjective Genetic Algorithm.

I. INTRODUCTION

A control is robust when the adjustment mechanism of the controller takes account of certain classes of parametric uncertainties and dynamics of the plant to be controlled. The controller is designed to make the control system stable, in spite of the uncertainties or parametric changes in the plant to be controlled, so as to establish tracking of the reference trajectory within the frequency range of interest. Since designers and engineers need to deal with industrial plants increasingly complex, taking into account structural and dynamic features such as nonlinearities, uncertainties, parametric variations, time delay, among others, several methods of robust control has been proposed, allowing in their formulation the use of constraints and performance requirements [1][2][3][4][5][6]. In [7], the stabilization of a discrete time robust control system based on reference model, is achieved. In [8], the design of robust control for perturbed systems, is presented. In [9], a fuzzy PID control to improve the stability of hydraulic position servo system, is proposed. Furthermore, fuzzy controllers have been a good alternative for control of complex dynamic systems, once that the fuzzy structure based on rules

are able to treat uncertainties, nonlinearities and time delay problems [10][11][12][13].In this paper, a model based robust fuzzy digital PID control strategy, via multiobjective genetic algorithm, based on the gain and phase margins specifications with applications to uncertain dynamic systems with time delay, is proposed.

II. STRATEGY FOR ROBUST FUZZY DIGITAL PID CONTROLLER DESIGN

A. Tuning formulas for model based control via gain and phase margins specifications

The Takagi-Sugeno (TS) fuzzy inference system to be used as model of the uncertain dynamic system, presents the $i|^{[i=1,2,...,l]}$-th rule given by:

$R^{(i)}$: IF $\widetilde{y}(k-1)$ IS $F^i_{k|\widetilde{y}(k-1)}$ THEN

$$G^i_P(z) = \frac{K^i_1 z + K^i_2}{a^i z^2 + b^i z + c^i}$$

where a^i, b^i, c^i, K^i_1, and K^i_2 are the parameters to be estimated by least square algorithm. The variable $\widetilde{y}(k-1)|^{[t=1,2,...,n]}$ belongs to fuzzy set $F^i_{k|\widetilde{y}(k-1)}$ with a value $\mu^i_{F_{k|\widetilde{y}(k-1)}}$ defined by a membership function $\mu^i_{\widetilde{y}(k-1)} : R \rightarrow [0,1]$, with $\mu^i_{F_{k|\widetilde{y}(k-1)}} \in \mu^i_{F_{1|\widetilde{y}(k-1)}}, \mu^i_{F_{2|\widetilde{y}(k-1)}}, \mu^i_{F_{3|\widetilde{y}(k-1)}}, \cdots , \mu^i_{F_{\widetilde{y}(k-1)|\widetilde{y}(k-1)}}$, where $p_{\widetilde{y}(k-1)}$ is the partitions number of the universe of discourse related to linguistic variable $\widetilde{y}(k-1)$ [14] [15][16]. The TS fuzzy digital PID controller, according to Parallel Distributed Compensation (PDC), presents the $j|^{[j=1,2,...,l]}$-th rule given by:

$R^{(j)}$: IF $\widetilde{y}(k-1)$ IS $F^j_{k|\widetilde{y}(k-1)}$ THEN

$$G^j_c(z) = \frac{\alpha^j z^2 + \beta^j z + \gamma^j}{z^2 + z}$$

where:

$$\alpha^j = K^j_P + \frac{K^j_I T}{2} + \frac{K^j_D}{T} \tag{1}$$

$$\beta^j = \frac{K^j_I T}{2} - K^j_P - \frac{2K^j_D}{T} \tag{2}$$

$$\gamma^j = \frac{K^j_D}{T} \tag{3}$$

where K_P^j, K_I^j, and K_D^j are proportional, integral, and derivative fuzzy controller gains, and T is the sample time, respectively.

In the direct path of closed-loop control system, considering the TS fuzzy model, the time delay z^{-n}, and the fuzzy digital PID controller, it has:

$$G_c(\widetilde{y}(k-1),z)G_p(\widetilde{y}(k-1),z) = \sum_{j=1}^{l}\sum_{i=1}^{l}\gamma_i(\widetilde{y}(k-1))\times$$

$$\times\gamma_j(\widetilde{y}(k-1))\times\left(\frac{\alpha^j z^2+\beta^j z+\gamma^j}{z^2+z}\right)\times$$

$$\times\frac{K_1^i z+K_2^i}{a^i z^2+b^i z+c^i}z^{-n} \qquad (4)$$

The gain and phase margins of the fuzzy control system are given by:

$$arg[G_c(\widetilde{y}(k-1),e^{j\omega_p})G_P(\widetilde{y}(k-1),e^{j\omega_p})] = -\pi \qquad (5)$$

$$A_m = \frac{1}{|G_c(\widetilde{y}(k-1),e^{j\omega_p})G_P(\widetilde{y}(k-1),e^{j\omega_p})|} \qquad (6)$$

$$|G_c(\widetilde{y}(k-1),e^{j\omega_g})G_P(\widetilde{y}(k-1),e^{j\omega_g})| = 1 \qquad (7)$$

$$\phi_m = arg[G_c(\widetilde{y}(k-1),e^{j\omega_g})G_P(\widetilde{y}(k-1),e^{j\omega_g})] + \pi \qquad (8)$$

where the gain margin is given by (5) and (6), and the phase margin is given by (7) and (8), respectively. The ω_p is called phase crossover frequency and ω_g is called gain crossover frequency [17][5].

B. Robust Stability Analysis

For the robust fuzzy PID digital controller design, based on the gain and phase margins specifications, the following Axiom and Theorems are proposed:

Axiom: *The linear sub-models $G_p^i(z)|^{[i=1,2,...,l]}$ of the plant, are necessarily of minimum phase, i.e., all poles of the characteristic equation are placed inside of the unit circle of the z-plane.*

Theorem 1: Each robust PID sub-controller, $G_c^j(z)|^{[i=1,2,...,l]}$, guarantees the gain and phase margins specifications for the linear sub-model, $G_p^i(z)|^{[i=1,2,...,l]}$ with $i=j$, of the plant to be controlled.

Proof. The proof of this theorem is not shown once there is no space enough in this paper.

Theorem 2: Each robust PID sub-controller $G_c^j(z)|^{[i=1,2,...,l]}$ guarantees the stability for all linear sub-models $G_p^i(z)|^{[i=1,2,...,l]}$ of the nonlinear plant to be controlled.

Proof. The proof of this theorem is not shown once there is no space enough in this paper.

C. Multiobjetive Genetic Strategy for Controller Tuning

The GA proposed in this paper to optimize the parameters K_P^j, K_I^j, and K_D^j of fuzzy digital PID controller in the j-th rule from the gain and phase margins specifications, presents the cost function given by:

$$Cost = |A_{mcal} - A_{mesp}| + |P_{mcal} - P_{mesp}| \qquad (9)$$

where A_{mcal} and A_{mesp} correspond to the gain margin computed and specified, respectively; P_{mcal} and P_{mesp} correspond to the phase margin computed and specified, respectively. The crossover between two chromosomes, used by the multiobjective genetic algorithm generates two new offspring by a simple crossover operator, which performs a weighted sum between the genes of the parents in order to generate the offspring, as follow:

$$\begin{aligned} chromosome_1 &= [p_{m1}, p_{m2}, p_{m3}..., p_{mn}] \\ chromosome_2 &= [p_{p1}, p_{p2}, p_{p3}..., p_{pn}] \\ d_{new1} &= \beta * p_{mn} + (1-\beta) * p_{pn} \\ d_{new2} &= \beta * p_{pn} + (1-\beta) * p_{mn} \end{aligned} \qquad (10)$$

where the terms p_{mn} and p_{pn} represent the n genes mother chromosome ($chromosome_1$) and genes father chromosome ($chromosome_2$) respectively, d_{new} is new offspring generated from two chromosomes and β is a random value between 0 and 1. The mutation operator used in this paper selects randomly a gene from the chromosome of the population and change its value to any other, within the range of possible values that the gains of the fuzzy controller can take. The best chromosome of the previous generation is keeped for the next generation, which is complemented by selected parents and the result of the mutation on the offspring. The stages of evaluation, classification, partner selection, crossover, mutation, and formation of the new population are repeated at each iteration of the algorithm [18][19].

III. EXPERIMENTAL RESULTS

A. Description of the Plataform for Real Time Control of a Termic Process

The plataform for real time control is composed of the termic process, the software LabVIEW, the CompactRIO 9073, the analog input module NI 9219, the analog output module NI 9263, the temperature sensor LM 35 and the actuator based on CI TCA 785. The termic process consists in an AC 220Volts monophasic toaster, with functional temperature in the interval from $25°C$ to $265°C$. The Labview is a graphical programming language in which the supervisory system will be developed for real time analysis of the closed loop control: the temperature and input voltage signals will be received from the sensor through the data acquisition system, the fuzzy robust PID controller will be implemented to process the con-trol signal to be sent to the termic process. The plataform control system of the termic process for real time is shown in the Fig. 1.
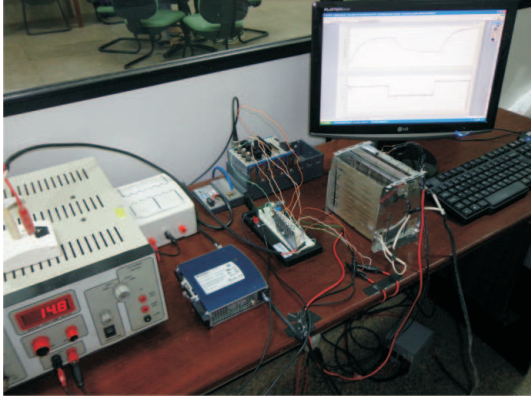
Fig. 1.   Plataform Control System of the Termic Process

Fig. 3.   Sample Cross Correlation Function between Input and Output Signals

B. TS Fuzzy Modeling for Termic Process

In the identification procedure, it was used the input and output signals of the uncertain dynamic system, as shown in Fig. 2. The time delay was estimated by computing sample cross-correlation function between the input and the output signals, resulting in a time delay of 1360 samples, considering the sample time of $17ms$, corresponding to 2.312 seconds [20], as shown in Fig. 3.

Fig. 2.   System Identification - Input and Output Signals

In order to estimate the fuzzy sets, the FCM (Fuzzy C-Means) algorithm was implemented for 2 clusters, weighting exponent *m=1.2* and tolerance *ϵ=0.01*. The fuzzy sets obtained are shown in Fig. 4. From the data input and output of the uncertain dynamic system, taking into account the weights of fuzzy sets, the least squares algorithm was applied for parameter estimation of the consequent sub-models, resulting in the fuzzy model structure of the uncertain dynamic system to be controlled. However, the identified TS fuzzy model was submitted to optimization procedure by genetic algorithm to improve DC gains to sub-models. The gains obtained were $g_1 = 0.0055$ and $g_2 = 0.72$, corresponding to sub-models G_p^1 and G_p^2, respectively. A comparative analysis for the temporal response performance of the termic process, TS fuzzy model optimized and identified is shown in Fig. 5. The identified TS fuzzy model after optimization procedure is given by (12).
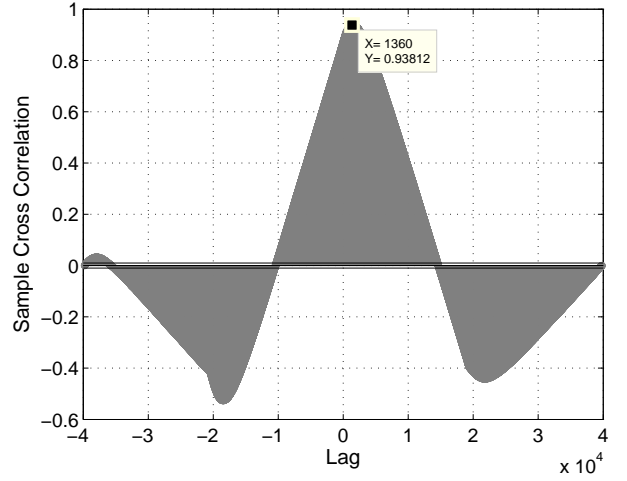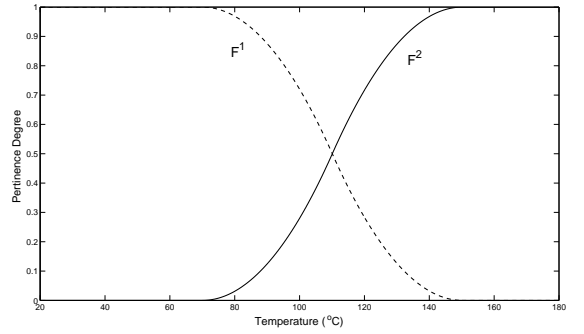

Fig. 4.   Membership Functions - FCM

$$R^1 : IF\ Temperature\ is\ F^1\ ,\ THEN$$

$$G_p^1(z) = 0.0055\frac{0.0513z + 0.0504}{z^2 - 0.5815z - 0.4177}z^{-136} \tag{11}$$

$$R^2 : IF\ Temperature\ is\ F^2\ ,\ THEN$$

$$G_p^2(z) = 0.72\frac{0.1344z - 0.1334}{z^2 - 0.5921z - 0.4072}z^{-136} \tag{12}$$

where:

$$F^1(a,b)|^{a=70;b=150.2} \tag{13}$$

$$\begin{cases} 1, Temperature \leq a \\ 1 - 2\left(\frac{Temperature-a}{b-a}\right)^2, a \leq Temperature \leq \frac{a+b}{2} \\ 2\left(b - \frac{Temperature}{b-a}\right)^2, \frac{a+b}{2} \leq Temperature \leq b \\ 0, Temperature \geq b \end{cases} \tag{14}$$

and $F^2 = 1 - F^1$.

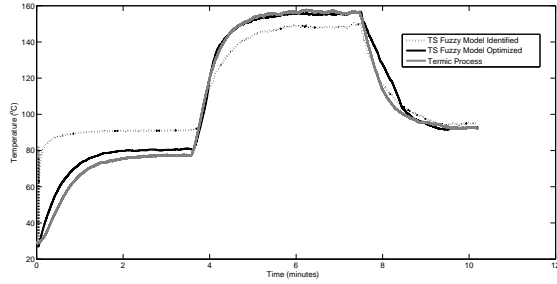From the multiobjetive genetic strategy proposed in this paper, specifying the appropriate gain and phase margins for

Fig. 5. Temporal Response Performance of the Termic Process, TS Fuzzy Model Optimized and TS Fuzzy Model Identified

Fig. 7. Bode Diagram of $G_p^2 G_c^2 z^{-\tau/T}$

the fuzzy control system, and considering the fuzzy model of the uncertain dynamic system with time delay of $n = 136$, the parameters of the fuzzy digital PID controller were obtained, according to table. I. It can be seen the eficiency of the proposed methodology in the model based PID controller design, since the gain and phase margins obtained of the fuzzy control system keeping closed of the specified ones. The Bode Diagram of $G_p^1 G_c^1$ and $G_p^2 G_c^2$ are shown in Fig. 6 and Fig. 7, respectively.

point was of 100^oC. The changing to 150^oC in the temperature set point was applied at 425 seconds. A gain variation was applied to the termic process at 212 seconds.

TABLE I
PARAMETERS OF PID CONTROLLER AND GAIN AND PHASE MARGINS
OBTAINED TO SUB-MODELS

Sub-model	(A_{me}, P_{me})	(A_{mc}, P_{mc})	PID Gains (K_P, K_I, K_D)
G_p^1	$(2.5, 60^\circ)$	$(2.9643, 63.8115^\circ)$	$(10, 0.15, 0.004)$
G_p^2	$(2.5, 60^\circ)$	$(2.5936, 62.6088^\circ)$	$(2.2178, 0.3176, 0.0013)$

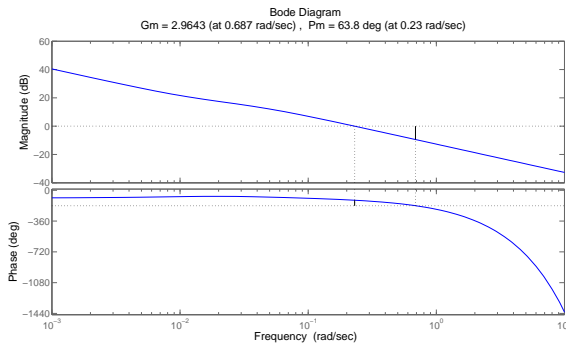Fig. 8. The cost of the best individual in each generation

Fig. 6. Bode Diagram of $G_p^1 G_c^1 z^{-\tau/T}$

The multiobjective genetic algorithm used the following parameters: 300 generations, random initial population of 100 chromosomes, each chromosome comprising three genes, the selection rate of 50% and the mutation rate of 12%. The performance of the multiobjetive genetic strategy, to minimize the multiobjetive cost function according to the number of generations, is shown in Fig. 8. The parameters of fuzzy digital PID controller obtained by the multiobjetive genetic strategy proposed was compared to Ziegler-Nichols Method [21]. The temporal response performance of the proposed methodology is shown in Fig. 9. The initial condition for temperature set
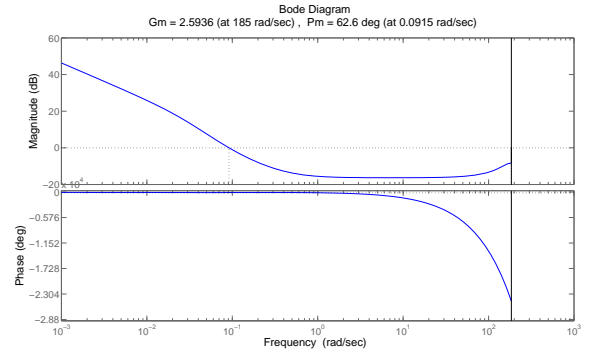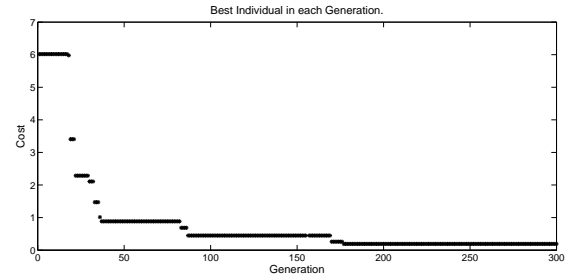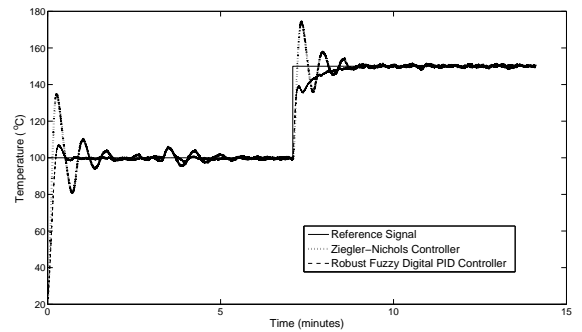

Fig. 9. Temporal Response Performance of the Ziegler-Nichols Controller and Robust Fuzzy PID Controller

Control action obtained with the robust fuzzy digital PID controller and de Ziegler-Nichols controller, and instantaneous gain and phase margins obtained with the robust fuzzy PID controller, as shown in Fig. 10 and Fig. 11.

IV. CONCLUSION

In this paper, a methodology for robust fuzzy digital PID control design of the uncertain dynamic systems with time delay, was proposed. The control scheme was eficient to ensure, through the multiobjetive genetic approach, since gain
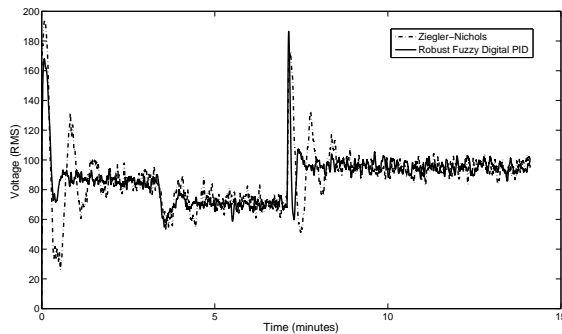
Fig. 10. Control action obtained with the robust fuzzy digital PID controller and the Ziegler-Nichols controller
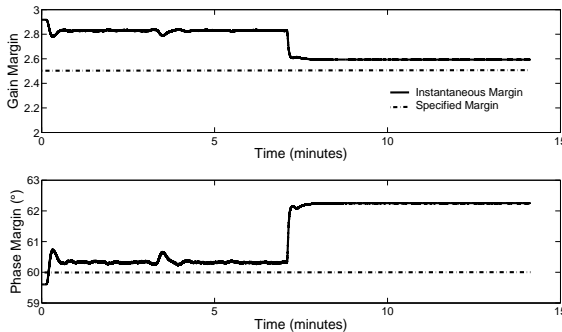

Fig. 11. Instantaneous gain and phase margins obtained with the robust fuzzy PID controller

and phase margins specified were get and keeping closed of the specified ones - despite the genetic algorithm application requires time and computational effort - as well as the tracking of the reference trajectory, even if with disturbance. The stability and high performance in the time has been guaranteed by the proposed TS fuzzy robust digital PID controller as compared with other controller from literature.

REFERENCES

[1] M. R. Matausek and T. B. Sekara, *PID controller frequency-domain tuning for stable, integrating and unstable processes, including dead-time*. Journal of Process Control 21:17-27, 2011.
[2] Q. C. Zhong, *Robust Control of Time-delay Systems*. British Library, USA, 2006.
[3] I. Y. A. Ozdemir, *Design of model-reference discrete-time sliding mode power system stabilizer*. International Workshop on Variable Structure Systems:325-330, 2008.
[4] Y. F. Peng, *Development of robust intelligent tracking control system for uncertain nonlinear systems using H∞ control technique*. Applied Soft Computing 11 3, April:3135-3146, 2011.
[5] M. E. Sezer and D. D. Siljak, *Robustness of suboptimal control: Gain and phase margin*. IEEE Transactions on Automatic Control 26:907-911, 1981.
[6] G. L. O. Serra, *Frontiers in Advanced Control Systems*. Rijeka:InTech, 2012.
[7] Y. Okuyama, *Model-reference type discretized PID control for continuous plants*. Proceedings of SICE Annual Conference:1402-1405, 2010.
[8] P. Pahuja and H. M. Rai, *Design of Robust Control for Perturbed Systems using QFT*. Second Internacional Conference on Advanced Computing and Communication Technologies:219-224, 2012.
[9] Y. Lin-ke, Z. Jian-ming, Y. Qi-long, X. Ji-ming and L. Yan, *Fuzzy PID Control for Direct drive Electro-hydraulic Position Servo System*. International Conference on Consumer Electronics, Communications and Networks:370-373, 2011.
[10] W. Wang and W. Lin, *Descentralized PDC for large-scale TS fuzzy systems*. IEEE Transactions on Fuzzy Systems:779-786, 2011.
[11] H. A. Hassan and M. P. R. V.RAO, *Research on direct torque control of induction motor based on genetic algorithm and Fuzzy adaptive PI Controller*. The International Conference on Computer as a Tool, vol.1:290-293, 2005.
[12] H. Li and C. Pan and F. Teng, *Adaptive Fuzzy Control based on Genetic Algorithm for Vertical Electric Furnace*. Proceedings of the IEEE International Conference on Automation and Logistics:933-938, 2009.
[13] L. H. Chen and C. H. Chiang and J. Yuan, *New approach to adaptive control architecture based on fuzzy neural network and genetic algorithm*. IEEE International Conference on Systems, Man and Cybernetics, vol.1:347-352, 2001.
[14] X. Cheng, Z. Lei and Y. Junqiu, *Fuzzy PID Controller for Wind Turbines*. 2nd International Conference on Intelligent Net-works and Intelligent Systems:74-77, 2009.
[15] C. H. Lee and C. C. Teng, *Tuning of PID controllers for stable and unstable processed based on gain and phase margin specifications: A fuzzy neural approach*. Fuzzy Sets and Systems 128:95-106, 2002.
[16] M. Petrov and I. Ganchev and A. Taneva, *Fuzzy PID control of nonlinear plants*. First International IEEE Symposium Intelligent Systems:30-35, 2002.
[17] Q. G. Wang and Z. Ye and C. C. Hang, *Tuning of phase-lead compensators for exact gain and phase margins*. Automatica 42:349-352, 2006.
[18] R. L. Haupt and S. E. Haupt, *Practical Genetic Algorithms*. John Wiley & Sons, New Jersey, 2004.
[19] D. S. Pereira and J. O. P. Pinto, *Genetic algorithm based system identification and PID tuning for optimum adaptive control*. Proceedings of the IEEE/ASME International Conference on Advanced Intelligent Mechatronics:801-806, 2005.
[20] L. A. Aguirre, *Introdução à Identificação de Sistemas: Técnicas Lineares e Não-Lineares Aplicadas a Sistemas Reais*. edition 3, UFMG, 2007.
[21] R. G. Jacquot, *Modern Digital Control Systems*. Marcel Dekker, 1995.