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Positional Control of a Flexible Structure using Neural Networks

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Abstract

This paper presents an adaptive inverse control approach for the positional control of an unconstrained multibody system with flexible appendages. The approach is called Feedback-Error-Learning *and it is based on the output of a feedback controller with fixed parameters to adapt a neural network which acts as a feedforward controller. The results are demonstrated by simulations using a high fidelity dynamic model of a experimental setup available at the ITA-IEMP Dynamics Laboratory.*

1. Introduction

The investigation of methodologies using artificial neural networks for control of lightweight materials with distributed flexibility in advanced space applications and in the construction of robotic manipulators has been the subject of intensive research in the recent years.

Based on their inherent learning ability and in the massively parallel architecture neural networks are considered promising controller candidates, particularly for nonlinear and uncertain systems. In most space applications, the dynamic behavior deviates considerably from the analytical model and the plant state cannot be physically measured or resolved without model dependent state estimation.

These characteristics have motivated the present research towards the development of neural network control methodologies that make use of real-time controller tuning and of output measurements. This effort resulted in the adaptive inverse control approach called *Feedback-Error-Learning* ([1], [2]) that uses the output of a previously adjusted feedback controller with fixed parameters to adapt a neural network which acts as a feedforward controller.

This paper describes the analytical model of a unconstrained multibody system ([3],[4]) and the adaptive control approach. The simulations of the positional control with potentiometer feedback of the experimental setup at the ITA-IEMP Dynamics Laboratory are also presented.

2. Analytical Models

2.1. Modelling the Unconstrained Multibody System

A schematic view of the experimental setup is shown below in figure 1. The unconstrained system under consideration is composed of two flexible appendages attached to a rigid hub and is driven by a brushless DC motor. The sensors used are a tachometer and a potentiometer which measure the hub angular velocity and position, respectively.

The unconstrained characteristic results from the natural motion without external influences, i.e., all the structure is allowed to vibrate and its solution involves both the inertia and the stiffness of the flexible parts.

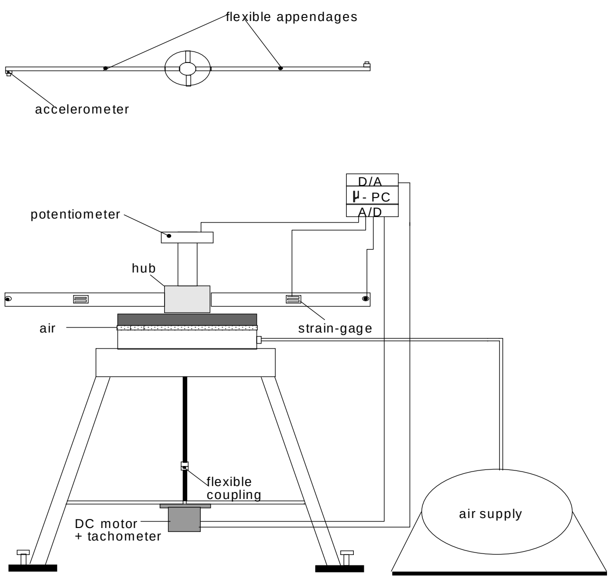

Figure 1 - Experimental setup

Applying Hamilton's principle and introducing the transformation of variables $z(x,t) = y(x,t) + x\theta(t)$, we determine the differential equations for this hybrid system:([3], [4])

$$\rho\,\ddot{z} + EI\,z^{iv} = 0 \qquad (1)$$

$$I_{hub}\ddot{\theta}+\rho\int_0^L x\ddot{z}dx+m_t L\ddot{z}_L=\tau \quad (2)$$

where L is the length of appendage, x is the position coordinate in the beam, y is the appendage deformation, θ is the constrained hub inertial rotation and the line (') and dot ($\cdot$) denote partial derivative with respect to space and time, respectively. The boundary conditions for a clamped-free system at $x=0$ and $x=L$ are:

$$z(0)=0\,,\ z'(0)=\theta \quad (3)$$

$$EIz''(L)=0\,,\ EIz'''(L)-m_{acel}\ddot{z}_L=0 \quad (4)$$

with the following momentum balance of the hub [3]:

$$EIz''(0)-I_{hub}\ddot{z}'(0)+\tau_m=0 \quad (5)$$

If we use: $\theta(t)=\theta\,cos(\omega t)$ and $z(x,t)=\phi(x)cos(\omega t)$, where θ is the modal amplitude of the rotational movement and $\phi(x)$ the unconstrained shapes function; in equation (1) then:

$$EI\phi^{iv}=\rho\phi\omega^2 \quad (6)$$

where $\phi^{iv}=\frac{\rho\omega^2}{EI}\phi$ and $\lambda^4=\frac{\rho\omega^2}{EI}$.

The above problem admits the possible unconstrained shape function:

$$\phi(x)=Asin(\lambda x)+B\cos(\lambda x)+Csinh(\lambda x)+D\cosh(\lambda x) \quad (7)$$

The constants A, B, C and D are chosen for normalization purposes. Evaluating the boundary conditions in equation (7), we obtain a set of homogeneous equations.

For a nontrivial solution, the determinant of the coefficients must vanish, given the following characteristic equation: [3]

$$\begin{aligned}\frac{2}{I_{hub}\lambda^3}\ &\left[I_{hub}\lambda^6(-1-cos(\lambda L)cosh(\lambda L))\right.\\&+\left(\gamma\lambda L\rho-\lambda^5 Lm_{acl}\right)sin(\lambda L)\\&+\left(\gamma I_{hub}\lambda^3-\lambda^3\rho\right)cosh(\lambda L)sin(\lambda L)\\&+\left(L\lambda^5 m_{acl}+\gamma L\lambda\rho\right)sinh(\lambda L)\\&+\left(\lambda^3\rho-\gamma I_{hub}\lambda^3\right)cos(\lambda L)sinh(\lambda L)\\&\left.-2\gamma\rho\,sin(\lambda L)sinh(\lambda L)\right]=0\end{aligned} \quad (8)$$

with the following orthogonality relationships:

$$EI\int_0^L\phi_r''\phi_s''\,dx=\omega_r^2\,\delta_{rs} \quad (9)$$

$$\rho\int_0^L\phi_r\phi_s\,dx+I_{hub}\phi_r'(0)\phi_s'(0)+m_{acel}\phi_r(L)\phi_s(L)=\delta_{rs} \quad (10)$$

The discrete model of the system is obtained by Ritz's Assumed Modes Method. In this method the elastic displacement can be described as:

$$y(x,t)=\sum_{j=1}^{N}\phi_j(x)\eta_j(t),\ 0\le x\le L \quad (11)$$

where $\phi(x)$ are unconstrained shape functions and $\eta(t)$ are time-varying coefficients. Applying Lagrange method in equations (1) and (2) and using the orthogonality relationships, the following matrix equation is obtained for the modes:

$$M\ \ddot{q}+K\ q=F \quad (12)$$

where:

$$M=\begin{bmatrix}I_\theta & M_{\eta1\theta}{}^T & M_{\eta2\theta}{}^T\\ M_{\eta1\theta} & M_{\eta1\eta1} & 0\\ M_{\eta2\theta} & 0 & M_{\eta2\eta2}\end{bmatrix} \quad (13)$$

$$K=\begin{bmatrix}0 & 0 & 0\\ 0 & K_{\eta1\eta1} & 0\\ 0 & 0 & K_{\eta2\eta2}\end{bmatrix} \quad (14)$$

$$q=[\theta\quad\eta_1\quad\eta_2]^T,\ F=[F_\theta\quad F_{\eta1}\quad F_{\eta2}]^T \quad (15)$$

with:

$$I_\theta=J_m+\sum_{i=1}^{2}\int_0^L m(x)\,(x+r)^2\,dx_i\ +m_t\,(L+r)^2 \quad (16)$$

$$M_{\eta_1\theta}=\int_0^L m(x)\,(x+r)\phi_1(x_1)dx_1+m_t\,(L+r)\phi_1(l) \quad (17)$$

$$M_{\eta_2\theta}=\int_0^L m(x)\,(x+r)\phi(x_2)dx_2 \quad (18)$$

$$M_{\eta_1\eta_1}=\int_0^L m\,(x)\,(x+r)\phi^2(x_1)dx_1\ +m_t(L+r)\phi^2(L) \quad (19)$$

$$M_{\eta_2\eta_2}=\int_0^L m(x)\ \phi^2(x_2)dx_2 \quad (20)$$

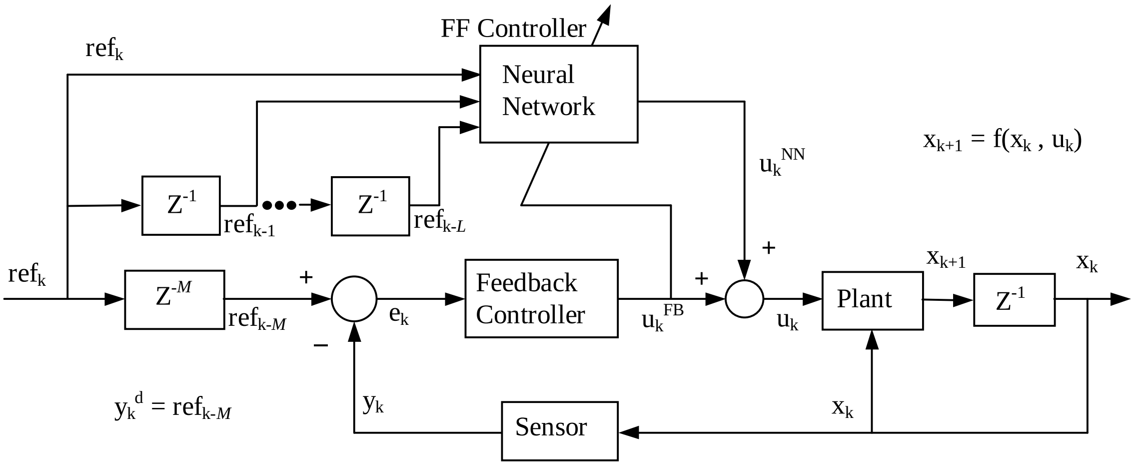

Figure 2 - Adaptive inverse control approach *Feedback-Error-Learning*

2.2. The Neural Network

The figure 2 shows the adaptive control approach used in this work. In relation with the approach proposed by Kawato *et al.* ([1], [2]) we introduced two modifications: a) the second order derivatives of the reference signal were replaced by a tapped delay line of length L, and b) a delay of M sampling periods were added to the reference signal.

Assuming that the plant is a linear dynamic system, stable, SISO (single input single output), time-invariant with transfer function in the discrete domain G(z) given by:

$$\begin{aligned} G(z) &= \frac{Y(z)}{U(z)} = \frac{b_0 z^{Nb} + b_1 z^{Nb-1} + \ldots + b_{Nb}}{a_0 z^{Na} + a_1 z^{Na-1} + a_2 z^{Na-2} + \ldots + a_{Na}} \\ &= \frac{B(z)}{A(z)} = \sum_{i=0}^{\infty} \beta_i z^{-i} = \beta_0 + \beta_1 z^{-1} + \beta_2 z^{-2} + \ldots \end{aligned} \tag{21}$$

where N_a and N_b are non-negative integers, and $N_a \geq N_b$.

Let's assume that the feedforward controller is a linear filter with finite impulse response [5], that is, a linear neural network. Therefore the transfer function of neural network will be given by:

$$G^{NN}(z) = \frac{U^{NN}(z)}{Ref(z)} = \alpha_0 + \alpha_1 z^{-1} + \ldots + \alpha_L z^{-L} \tag{22}$$

And from figure 1:

$$E(z) = z^{-M} Ref(z) - Y(z) \tag{23}$$

$$G^{FB}(z) = \frac{U^{FB}(z)}{E(z)} \tag{24}$$

$$U(z) = U^{FB}(z) + U^{NN}(z) \tag{25}$$

Combining this equations, we have:

$$\frac{Y(z)}{Ref(z)} = \frac{G(z)\left[z^{-M} G^{FB}(z) + G^{NN}(z)\right]}{1 + G(z)G^{FB}(z)} \tag{26}$$

$$\frac{U^{FB}}{Ref(z)} = \frac{G^{FB}(z)\left[z^{-M} - G(z)G^{NN}(z)\right]}{1 + G(z)G^{FB}(z)} \tag{27}$$

If $G^{NN}(z) = z^{-M} / G(z)$, then from eqs. (26) e (27):

$$\frac{Y(z)}{Ref(z)} = z^{-M} \tag{28}$$

$$\frac{U^{FB}(z)}{Ref(z)} = \frac{E(z)}{Ref(z)} = 0 \tag{29}$$

However, since it is assumed that the neural network $G^{NN}(z)$ has the structure defined by the equation (22) with a finite number (L+1) of coefficients, the neural network could be seen as a truncated representation of the pulse response of the inverse model of the plant delayed by M sample periods.

Assuming that the closed-loop pulse response of the plant when using just the feedback controller (without the neural network) is stable, then the polynomials $\gamma(z)$ e $\phi(z)$, defined below, converge:

$$\gamma(z) = \frac{G^{FB}(z)}{1 + G(z)G^{FB}(z)} = \sum_{j=0}^{\infty} \gamma_j z^{-j} \tag{30}$$

$$\phi(z) = \frac{G(z)G^{FB}(z)}{1 + G(z)G^{FB}(z)} = \sum_{j=0}^{\infty} \phi_j z^{-j} \tag{31}$$

Combining eqs. (30) e (31) with eq. (27):

$$\mathrm{U}^{\mathrm{FB}}(\mathrm{z}) = \left[\mathrm{z}^{-M}\gamma(\mathrm{z}) - \phi(\mathrm{z})\mathrm{G}^{\mathrm{NN}}(\mathrm{z})\right]\mathrm{Ref}(\mathrm{z}) \quad (32)$$

The estimation of the coefficients of the neural network $\mathrm{G}^{\mathrm{NN}}(\mathrm{z})$ in eq. (22) can be defined as an optimization problem. In this case it is desired to find the coefficients of neural network that minimize the mean value of the square of the feedback controller output, defined as the following scalar cost function *J*:

$$\begin{aligned} J &= \frac{1}{2}\mathrm{E}\left[\left(\mathrm{u}_{\mathrm{k}}^{\mathrm{FB}}\right)^2\right] \\ &= \frac{1}{2}\mathrm{E}\left[\left(\mathrm{z}^{-M}\gamma(\mathrm{z})\mathrm{ref}_{\mathrm{k}} - \phi(\mathrm{z})\left(\alpha^*\right)^{\mathrm{T}}\mathrm{ref}_{\mathrm{k}}^*\right)^2\right] \end{aligned} \quad (33)$$

where:

$$\alpha^* = \left[\alpha_0 \quad \alpha_1 \quad \ldots \quad \alpha_L\right]^{\mathrm{T}} \quad (34)$$

$$\mathrm{ref}_{\mathrm{k}}^* = \left[\mathrm{ref}_{\mathrm{k}} \quad \mathrm{ref}_{\mathrm{k}-1} \quad \ldots \quad \mathrm{ref}_{\mathrm{k}-L}\right]^{\mathrm{T}} \quad (35)$$

Notice that α^* e $\mathrm{ref}_{\mathrm{k}}^*$ are column vectors with $L+1$ components.

It is important to show, that under certain conditions: 1) the scalar cost function *J* has a unique minimum; 2) the parameters of the neural network that minimize the cost function *J* could be used as an approximation of the delayed inverse model of the plant $\mathrm{G}(\mathrm{z})$.

The stationary points of *J* are given by $\partial J/\partial\alpha^* = [0 \quad 0 \quad \ldots \quad 0]^{\mathrm{T}} = \mathbf{0}^{\mathrm{T}}$, where:

$$\frac{\partial J}{\partial \alpha^*} = \mathrm{E}\left[\mathrm{u}_{\mathrm{k}}^{\mathrm{FB}}\frac{\partial \mathrm{u}_{\mathrm{k}}^{\mathrm{FB}}}{\partial\alpha^*}\right] = \mathrm{E}\left[\mathrm{u}_{\mathrm{k}}^{\mathrm{FB}}\left(-\phi(\mathrm{z})\mathrm{ref}_{\mathrm{k}}^*\right)\right] = \mathbf{0}^{\mathrm{T}} \quad (36)$$

The cost function J will have minimum if the matrix $\partial^2 J/\partial\alpha^{*2}$ is positive definite, where:

$$\frac{\partial^2 J}{\partial\alpha^{*2}} = \mathrm{E}\left\{\left[\phi(\mathrm{z})\mathrm{ref}_{\mathrm{k}}^*\right]\left[\phi(\mathrm{z})\mathrm{ref}_{\mathrm{k}}^*\right]^{\mathrm{T}}\right\} \underline{\Delta} F^l \quad (37)$$

The matrix F^l is the correlation matrix of vector $\phi(\mathrm{z})\mathrm{ref}_{\mathrm{k}}^*$ with dimension *L*+1 by *L*+1. From eqs. (32) and (36):

$$F^l\alpha^* = F^r \quad (38)$$

where:

$$F^r = \mathrm{E}\left[\left(\mathrm{z}^{-M}\gamma(\mathrm{z})\mathrm{ref}_{\mathrm{k}}\right)\left(\phi(\mathrm{z})\mathrm{ref}_{\mathrm{k}}^*\right)\right] \quad (39)$$

where F^r is a column vector of *L*+1 rows. Considering the reference signal as stationary, i.e., $\mathrm{E}\left[\mathrm{ref}_{\mathrm{k}}\mathrm{ref}_{\mathrm{k}\pm\mathrm{i}}\right] = \rho_{\mathrm{i}} = \rho_{-\mathrm{i}}$, the elements (i', j') of F^l and the element (i') of F^r could be written as:

$$\begin{aligned} F_{\mathrm{i}'\mathrm{j}'}^l &= \mathrm{E}\left\{\left[\sum_{\mathrm{i}=0}^{\infty}\phi_{\mathrm{i}}\mathrm{ref}_{\mathrm{k}-\mathrm{i}-(\mathrm{i}'-1)}\right]\left[\sum_{\mathrm{j}=0}^{\infty}\phi_{\mathrm{j}}\mathrm{ref}_{\mathrm{k}-\mathrm{j}-(\mathrm{j}'-1)}\right]\right\} \\ &= \sum_{\mathrm{i}=0}^{\infty}\sum_{\mathrm{j}=0}^{\infty}\phi_{\mathrm{i}}\phi_{\mathrm{j}}\rho_{[\mathrm{i}-\mathrm{j}+\mathrm{i}'-\mathrm{j}']} \end{aligned} \quad (40)$$

$$\begin{aligned} F_{\mathrm{i}'}^r &= \mathrm{E}\left\{\left[\sum_{\mathrm{i}=0}^{\infty}\gamma_{\mathrm{i}}\mathrm{ref}_{\mathrm{k}-\mathrm{i}-M}\right]\left[\sum_{\mathrm{j}=0}^{\infty}\phi_{\mathrm{j}}\mathrm{ref}_{\mathrm{k}-\mathrm{j}-(\mathrm{i}'-1)}\right]\right\} \\ &= \sum_{\mathrm{i}=0}^{\infty}\sum_{\mathrm{j}=0}^{\infty}\gamma_{\mathrm{i}}\phi_{\mathrm{j}}\rho_{[\mathrm{i}-\mathrm{j}+M-(\mathrm{i}'-1)]} \end{aligned} \quad (41)$$

where $1 \le \mathrm{i}' \le L+1$ e $1 \le \mathrm{j}' \le L+1$. Notice that: 1) F^l is a symmetric matrix with the Toeplitz (banded) structure common to covariance matrices, and 2) eqs. (40) and (41) assume that the polynomials $\gamma(\mathrm{z})$ and $\phi(\mathrm{z})$ converge.

The linear system equation (38) will have a unique solution if the reference signal is exciting enough such that F^l is positive definite. The calculated neural network parameters α_{i} $(0 \le \mathrm{i} \le L)$ will be the set that minimizes the mean value of the square of the feedback controller output. The specific values of L and M will determine how small the minimum of the cost function *J* is.

However, the matrices F^r e F^l can only be evaluated if the transfer function of the plant G(z) is precisely known. If the transfer function G(z) is not precisely known or is slowly time-variant, we can use a learning algorithm to searches for the neural network parameters that minimize the cost function J.

2.3. The Feedback-Error-Learning Rule

Considering $\hat{\alpha}_{\mathrm{i}}(\mathrm{k})$ the estimated value of the neural network coefficient α_{i} at time step k, a possible simple approach is to use an algorithm like *gradient descent*. In this approach the neural network coefficients are changed in the direction that decreases the cost function J, that is:

$$\hat{\alpha}_{\mathrm{i}}(\mathrm{k}+1) = \hat{\alpha}_{\mathrm{i}}(\mathrm{k}) - \eta\left[\frac{\partial J}{\partial\alpha_{\mathrm{i}}}\right]_{\alpha^* = \hat{\alpha}^*(\mathrm{k})} \quad (42)$$

where η is the learning rate; i = 0, 1, ..., L; and:

$$\hat{\alpha}^*(\mathrm{k}) = [\hat{\alpha}_0(\mathrm{k}) \quad \hat{\alpha}_1(\mathrm{k}) \quad \dots \quad \hat{\alpha}_L(\mathrm{k})]^{\mathrm{T}} \qquad (43)$$

From eq. (16):

$$\hat{\alpha}_{\mathrm{i}}(\mathrm{k}+1) = \hat{\alpha}_{\mathrm{i}}(\mathrm{k}) + \eta\, \mathrm{u}_{\mathrm{k}}^{\mathrm{FB}} \sum_{\mathrm{j}=0}^{\infty} \phi_{\mathrm{j}}\, \mathrm{ref}_{\mathrm{k\text{-}i\text{-}j}} \qquad (44)$$

However the polynomial $\phi(\mathrm{z})$, the closed loop transfer function without the neural network, cannot be assumed to be known a priori, since it would imply the knowledge of G(z). Therefore we propose to replace the polynomial $\phi(\mathrm{z})$ by a guessed known polynomial $\lambda(\mathrm{z})$, where $\lambda(\mathrm{z})$ should be a **coarse approximation** of $\phi(\mathrm{z})$ [6]. So in this work, instead of using eq. (41) as our learning algorithm, we proposed to use the following rule, called **Feedback-Error-Learning Rule**:

$$\hat{\alpha}_{\mathrm{i}}(\mathrm{k}+1) = \hat{\alpha}_{\mathrm{i}}(\mathrm{k}) + \eta\, \mathrm{u}_{\mathrm{k}}^{\mathrm{FB}} \sum_{\mathrm{j}=0}^{NL} \lambda_{\mathrm{j}}\, \mathrm{ref}_{\mathrm{k-i-j}} \qquad (45)$$

where:

$$\lambda(\mathrm{z}) = \lambda_0 + \lambda_1 \mathrm{z}^{-1} + \dots + \lambda_{NL} \mathrm{z}^{-NL} \qquad (46)$$

$$\lambda^* = [\lambda_0 \quad \lambda_1 \quad \dots \quad \lambda_{NL}]^{\mathrm{T}} \qquad (47)$$

Our experiments show that in many cases we can simply use $\lambda^* = \lambda_0 = 1$, for example, in plants where the feedback controller was adjusted in such a manner that the polynomial $\phi(\mathrm{z})$ converge quickly without oscillations.

Note that, as the quality of the feedback controller increases, the approximation error introduced by using $\lambda(\mathrm{z})$ instead of $\phi(\mathrm{z})$ decreases, since $\phi(\mathrm{z})$ will converge more rapidly to zero.

3 - Numerical Simulations

In this section we show the simulation of the positional control of the unconstrained multibody system with potentiometer feedback. The input and output of the plant are respectively the applied torque F_θ and the angular position θ measured by the potentiometer. Therefore the plant transfer function is given as $\mathrm{G}(\mathrm{z}) = F_\theta(\mathrm{z}) / \theta(\mathrm{z})$.

The mass and the stiffness matrices of the unconstrained system, eqs. (13) and (14), were calculated using MATHEMATICA, a symbolic manipulator program:

$$M = \begin{bmatrix} 0.00767 & 0.0417 & 0.06255 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad (48)$$

$$K = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 5543.12066 & 0 \\ 0 & 0 & 34672.54922 \end{bmatrix} \qquad (49)$$

Assuming that the reference signal $\mathrm{ref}_{\mathrm{k}}$ is such that $\rho_{\mathrm{i}} = \rho_{-\mathrm{i}} = 0$ for $\mathrm{i} \geq 2$, we can rewrite the equations (40) e (41) as:

$$\mathrm{F}_{\mathrm{i'j'}}^{\mathrm{l}} = \tilde{n}_0 \sum_{\mathrm{i}=0}^{\infty} \phi_{\mathrm{i}} \phi_{\mathrm{i}+(\mathrm{i'}-\mathrm{j'})} + \tilde{n}_1 \sum_{\mathrm{i}=0}^{\infty} \phi_{\mathrm{i}} \left[\phi_{\mathrm{i}-1+(\mathrm{i'}-\mathrm{j'})} + \phi_{\mathrm{i}+1+(\mathrm{i'}-\mathrm{j'})}\right] \qquad (50)$$

$$\mathrm{F}_{\mathrm{i'}}^{\mathrm{r}} = \tilde{n}_0 \sum_{\mathrm{i}=0}^{\infty} \tilde{\mathrm{a}}_{\mathrm{i}} \phi_{\mathrm{i}-\mathrm{i'}+M+1} + \tilde{n}_1 \sum_{\mathrm{i}=0}^{\infty} \tilde{\mathrm{a}}_{\mathrm{i}} \left[\phi_{\mathrm{i}-\mathrm{i'}+M} + \phi_{\mathrm{i}-\mathrm{i'}+M+2}\right] \qquad (51)$$

where, by definition, $\phi_i = 0$ for $\mathrm{i} < 0$. This reference signal $\mathrm{ref}_{\mathrm{k}}$ was generated by:

$$\mathrm{ref}_{\mathrm{k}} = S_0 s_{\mathrm{k}} + S_1 s_{\mathrm{k}-1} \qquad (52)$$

where s_{k} is a white noise sequence uniformly distributed between –1 e 1. Setting $S_0 = 1$ and $S_1 = 0.7$, then, by definition:

$$\mathrm{E}[s_{\mathrm{k}} s_{\mathrm{k}\pm\mathrm{i}}] = \begin{cases} \rho_{\mathrm{s}} = \dfrac{1}{3}, \text{para i} = 0 \\ 0, \text{para i} \neq 0 \end{cases} \qquad (53)$$

$$\mathrm{E}[\mathrm{ref}_{\mathrm{k}} \mathrm{ref}_{\mathrm{k}\pm\mathrm{i}}] = \rho_{\mathrm{i}} = \begin{cases} (S_0^2 + S_1^2)\rho_{\mathrm{s}} = \dfrac{1.49}{3}, \text{se i} = 0 \\ S_0 S_1 \rho_{\mathrm{s}} = \dfrac{0.7}{3}, \text{se i} = 1 \\ 0, \text{se i} > 1 \end{cases} \qquad (54)$$

Using a PID controller as the feedback controller, then:

$$\mathrm{G}^{\mathrm{FB}}(\mathrm{z}) = \frac{(K_p + K_i + K_d)\mathrm{z}^2 - (K_p + 2\,K_d)\mathrm{z} + K_d}{\mathrm{z}^2 - \mathrm{z}} \qquad (55)$$

and the following gains were selected: $K_p = 1.0$, $K_i = 0.0$, $K_d = 9$. The parameters L and M were chosen respectively as 35 and 1.

The theoretical solution for the neural network parameters α^* can be calculated using eqs. (38), (50) and (51). The plant transfer function G(z) can be calculated using eq. (12).

The approach *Feedback-Error-Learning* was simulated during 2600 seconds, the first 2500 seconds were the **learning phase** of the neural network and the other 100 seconds were the **test phase**. The neural network coefficients were initialized as zero such that in the beginning of the learning phase only the feedback controller was used. During the learning phase the Feedback-Error-Learning Rule (eq. (45)) was applied using $\lambda^* = \lambda_0 = [0 \quad 1]$, $\eta = 0.105$, and the reference signal ref_k was generated as describe in eq. (52).

Figure 3 shows the error signal e_k (input of the feedback controller) during the learning and test phases. Figure 4 shows the theoretical neural network parameters α^* and their estimated values $\hat{a}^*$ (experimental solution) at the end of the learning phase.

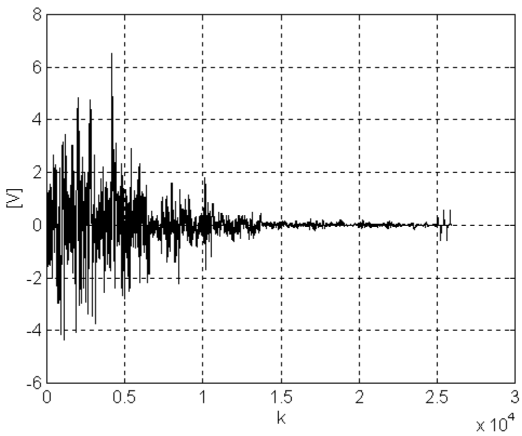

Figure 3 - The error signal e_k (input of the feedback controller) during the learning and test phases

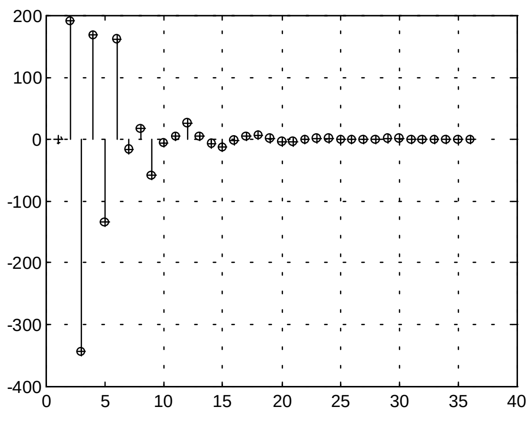

Figure 4 - The neural network parameters α^* (o) and their estimated values $\hat{a}^*$ (+)

4. Conclusions

This paper shows how to perform positional control of a unconstrained multibody system using neural networks trained by the Feedback-Error-Learning Rule. The simulation results show that small tracking errors and good convergence for the neural network parameters can be achieved. [6]

Rios Neto *et al.* [7] and Nascimento Jr. [8] show respectively how this neural control approach can be applied to non-minimum-phase linear and non-linear plants as well.

Future research will investigate the performance of this neural control approach when used to suppress vibration in real-time.

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