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A SURVEY ON THE NONLINEAR ANALYSIS OF QUANTUM DOT SEMICONDUCTOR LASERS

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Resumo – Lasers de pontos quânticos semicondutores constituem uma classe recente de fontes laser para uma gama de aplicações desde comunicações ópticas até instrumentação biomédica, e aparece como uma promissora alternativa aos lasers de poços quânticos por causa de importante propriedades como, por exemplo, a reduzida corrente de limiar, a grande largura de banda do ganho e o alto ganho diferencial. A sensibilidade à realimentação óptica também é um importante aspecto de tais dispositivos e tem atraído muita atenção ultimamente, pois relevantes questões de projeto podem ser melhoradas, com significativa economia sendo esperada. Além disso, estes dispositivos apresentam rica variedade de comportamento dinâmico para o qual muito frequentemente a base matemática de sistemas não-lineares se faz necessária. Para oferecer aos leitores uma introdução à modelagem destes dispositivos, neste artigo uma pesquisa sobre os lasers de pontos quânticos é apresentada, com ênfase nos progressos da literatura dos últimos 10 anos. Isto inclui a caracterização da dinâmica caótica através dos expoentes de Lyapunov, a avaliação da dimensão de atratores estranhos, a determinação de rotas para o caos a partir de diagramas de bifurcação e outras abordagens da dinâmica não-linear de sistemas.

Palavras-chave – Caracterização de caos, expoentes de Lyapunov, diagramas de bifurcação, lasers de pontos quânticos.

Abstract – Quantum dot semiconductor lasers constitute a recent type of laser sources for a variety of applications ranging from optical communications to biomedical instrumentation, and it appears as a prominent alternative over the precedent quantum well lasers because of important properties as, for instance, the low threshold current, the high gain bandwidth and the high differential gain. The sensitivity to optical feedback is also an important feature of such devices and have attracted a lot of attention in the last decades because relevant design issues can be improved, with significant cost saving expected. Furthermore, these devices present a very rich variety of dynamic behavior for which very often the mathematical framework of nonlinear systems is required. To offer readers an introduction to the modeling of these devices, in this paper a survey on the quantum dot lasers is presented, with emphasis on the achievements of the literature in the last 10 years. This includes the characterization of chaotic dynamics through the calculation of Lyapunov exponents, the evaluation of the dimension of strange attractors, the determination of routes to chaos from bifurcation diagram and other approaches from nonlinear system dynamics.

Keywords – Chaos characterization, Lyapunov exponents, bifurcation diagrams, quantum dot lasers.

1. INTRODUCTION

The last decades have seen enormous progress in the field of optoelectronic devices, with the development of light sources, detectors, amplifiers and modulators optimized for a wide range of applications in telecommunications, information processing and in medicine, as well [1].

Such evident progress can be mainly attributed to the enhancement of semiconductor growth techniques in the years. In fact, in the 60's they played a fundamental role in the development of semiconductor heterostructures, which represented, in turn, an essential step towards the improvement of the performance of existent laser sources, with CW laser operation regime [2] and threshold current reduction being the main breakthroughs.

In its simplest configuration, a semiconductor laser source looks like depicted in Figure 1. It is essentially a two-paralleled mirror structure able to confine photons in an optical cavity; the cavity is, in turn, made by an active material able to provide optical gain from light-matter interaction, meaning that the optical field intensity increase as light propagates in the cavity. This kind of device is called edge-emitting laser because light leaves the cavity from the lateral interface in which the lower reflectivity mirror is placed (in the figure, $R_2 < R_1$).

Additional progress was achieved in the middle 70's, with the exploitation of quantum effects in semiconductor heterostructure lasers as a way to achieve wavelength tunability and to reduce even more the threshold current [3]. This seminal paper in the field of optoelectronics showed the advantages of quantum well (QW) lasers over the conventional ones (bulk) and, still more important, it gave an indication of the improved lasers that could be fabricated by further exploiting the reduced dimensionality of the devices, which later would be referred to as quantum wire (2D-confinement) and, ultimately, quantum dot (3D-confinement) lasers (QD lasers).

Devices named QD lasers present in their active region nanosized islands of semiconductor material (quantum dots) able to confine carriers in all three spatial directions; such a feature gives them a number of advantages over the other alternatives of

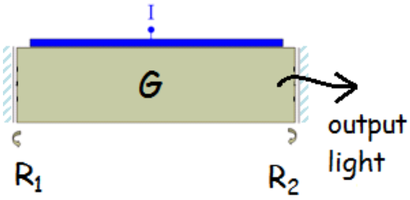


Figura 1: Edge-emitting semiconductor laser structure

semiconductor laser sources. Since the goal of the present paper is not to enter into the details of the physics of semiconductor lasers (for a detailed study see e.g. section 5.8.7 in [4]), it is enough to mention that important operating characteristics like modulation bandwidth, frequency chirp and damping rate are favored in ideal QD lasers [5,6]. All those characterstics are indeed influenced by the differential gain, a parameter which indicates how the gain in the active material varies with carrier density change and is expressed as $\frac{dg}{dn}$. The peculiarity of a quantum dot laser source is that the differential gain is very high in these devices, which in turn leads to low chirp, low linewidth enhancement factor and high damping rate if compared to the precedent quantum well and/or bulk lasers.

In practice, however, quantum dot samples suffer from a number of issues related to the growth proces, which ultimately limit their expected performance as active material for fast, robust and stable laser sources, thus motivating much work worldwide on the study of the dynamic behaviour of QD lasers in the recent years [7–10]. It is worth mentioning that dynamic behaviour is here understood as the time-series response to different current injection conditions (and at different operating conditions) and, since a laser source is a very typical nonlinear system (by the way widely used in basics nonlinear dynamics textbooks), studying its dynamics means using the concepts and tools of this field to interpret results.

Purpose of this paper is to review the field of nonlinear analysis of quantum dot laser dynamics, by reporting on (and linking) the previous and recent progresses achieved by researchers in the last 10 years. Initially the basics of semiconductor lasers is presented. This includes relevant equations and discussion about important operating issues. Then some concepts of nonlinear dynamic systems theory are recalled, especially those necessary for understanding the results of the literature regarding the characterization of chaotic dynamics as, for instance, the idea of strange attractor (and the reconstruction theorem) and the calculation of Lyapunov exponents, as well as the route to chaos from power spectrum and bifurcation diagram. After that, the survey itself takes place, and papers and communications from relevant vehicles are described. Finally, some highlights for the near future of nonlinear analysis of QD laser devices are given in the conclusion.

2. BASICS OF QD LASER MODELING

In order to give, as fast as possible, good predictions of the expected behavior of a real QD laser device, many numerical models have been proposed in the literature to study its static and dynamic characteristics and also to investigate the carrier recombination mechanisms in the quantum dots [11]; these models are either based on the master equations approach [12, 13], which requires many-body theory to correctly describe the quantum dot microscopic transitions [14–16], or based on rate equations (RE), which assume the intraband relaxation is rapid enough to guarantee the carriers in the active region are in thermodynamic equilibrium [17, 18]. This is a very good assumption for laser operating at room-temperature and, thus, RE approach applies well to a wide range of applications. Furthermore, given its complexity, many-body theory based models inevitably leads to very slow computer codes, whereas the alternative rate equations based computer programs are simpler, faster and present very good agreement between simulation and experimental results of QD devices [19].

In the applications domain, an important use of QD lasers is in optical networks, as transmitter. In traditional optical communication setups, the laser source is followed by an optical isolator, a costy device which prevents back reflections to enter the device. According to [20] fed back light coming into a distributed feedback laser (DFB) cavity may lead the device to enter different operations regimes. In Regime I, there is very small feedback (less than 0.01% of the output power enters back the active region of the internal cavity) and very small effects like short broadening of the linewidth are observed. As feedback intensity increases, the device may enter Regime II, characterized by the appearance of external modes and mode hopping among internal and external modes. Mode hopping is suppressed around 0.1% of feedback, and narrow linewidth dominates the laser spectrum; this is Regime III. Moderate feedback (or Regime IV) occurs when feedback intensity reaches 1.0%, and produces significant linewidth broadening; additionally, chaotic behavior and coherence collapse results from unstable oscillations and rising of noise level. Last region is the strong feedback Regime V, in which 10.0% or higher intensity takes place; in this regime external and internal cavity behave like a single structure and the device oscillates in a single mode, with a narrow linewdith.

In view of efficient and mininum optical network design, it is therefore desirable to have a laser source robust to optical feedback. To this end, a number of experiments in QD lasers have been carried out. Most of them aims at finding the operating conditions (current, temperature, etc) for which the device is less sensitivity to external incoming light [21, 22]; studies and measurements performed to determine the dependence of the sensitivity on the device length have also given important input to the design of feedback-robust devices [23, 24]. For what concerns the modeling, the scientific community usually considers the very simple external cavity configuration as shown in Figure 2 associated to the classical Lang-Kobayashi rate-equations model [25] to study the laser performance in optical feedback scenario.

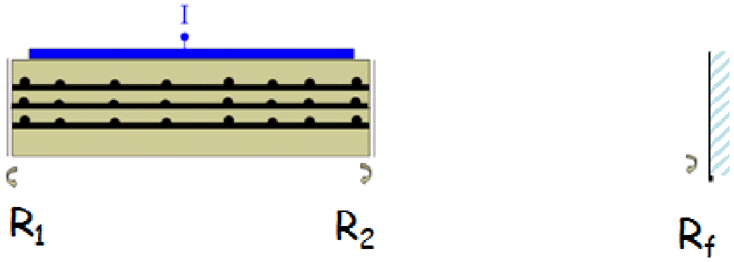


Figura 2: External cavity configuration used to model the optical feedback in quantum dot lasers. R_F represents the power reflectivity of the external mirror which reflects back the light scattered from the laser output facet, R_2

The interaction between photons and carriers in the cavity can be described by the following equations, which are based on the model of section 3.3 in [26] and [27]:

$$\dot{n}_w = \frac{n_s}{t_{sw}} + \frac{n_2}{t_{2w}} - \frac{n_w}{t_{ws}} - \frac{n_w \cdot (1-\rho_2)}{t_{w2}} - \frac{n_w \cdot (1-\rho_1)}{t_{w1}} - \frac{n_w \cdot (1-\rho_0)}{t_{w0}} \tag{1}$$

$$\dot{n}_2 = \frac{n_w \cdot (1-\rho_2)}{t_{w2}} + \frac{n_1 \cdot (1-\rho_2)}{t_{12}} - \frac{n_2 \cdot (1-\rho_1)}{t_{21}} - \frac{n_2}{t_{2w}} - R_{st2} \tag{2}$$

$$\dot{n}_1 = \frac{n_2 \cdot (1-\rho_1)}{t_{21}} + \frac{n_0 \cdot (1-\rho_1)}{t_{01}} + \frac{n_w \cdot (1-\rho_1)}{t_{w1}} - \frac{n_1 \cdot (1-\rho_2)}{t_{12}} - \frac{n_1 \cdot (1-\rho_0)}{t_{10}} - R_{st1} \tag{3}$$

$$\dot{n}_0 = \frac{n_1 \cdot (1-\rho_0)}{t_{10}} + \frac{n_w \cdot (1-\rho_0)}{t_{w0}} - \frac{n_0 \cdot (1-\rho_1)}{t_{01}} - R_{st0} \tag{4}$$

$$\dot{n}_s = \frac{I}{e} - \frac{n_s}{t_{sd}} + \frac{n_d}{t_{ds}} \tag{5}$$

$$\dot{n}_d = \frac{n_s}{t_{sd}} - \frac{n_d}{t_{ds}} \tag{6}$$

$$\dot{E}_0 = \frac{-E_0}{2t_{ph}} + \frac{c}{2n_r} - (g_{ES2} + g_{ES1} + g_{GS}) \cdot E_0 + k \cdot E_0 \left(t - t_d\right) \cdot cos\left(\omega_0 t_d + \Delta\right) + \beta_{sp} \tag{7}$$

$$\dot{\Phi} = 2\pi\delta f - \frac{k \cdot E_0 \left(t - t_d\right)}{E_0} \cdot sin\left(\omega_0 t_d + \Delta\right) \tag{8}$$

In these equations, the occurrence of the various scattering mechanisms have been taken according to time constants t_{if}, where i stands for the initial state and f for final state; t_{w2}, for instance, is an average estimate for the scattering time interval between the **w**etting layer and the excited state number **2**. The Pauli blocking phenomenon is taken into account by means of the occupation probabilities $\rho_k, k = 0, 1, 2$, which represent the average filling of the confined states (available states in the wetting layer is assumed to be infinite). Equations 7 and 8, which give the amplitude and phase of the in-cavity electrical field follow the Lang-Kobayashi model [25], but here the gain term (the sum of g_{ES2}, g_{ES1} and g_{GS}) is due to quantum-dot material, and couples photons and carriers in the cavity according to a Lorentzian-like linewidth broadening function. In the carrier equations, this coupling appears as a loss of electrons in the terms stimulated emission rates $R_{st2}, R_{st1}, R_{st0}$, which are in turn dependent on $|E_0|^2$(see [26] for details).

Finally, in these equations the terms $t_{ph}, c, n_r, k, t_d, \omega_0, \Delta, \delta f$ represent respectively the photon lifetime in the cavity, the light velocity in the free-space, the refractive index of the active medium, the feedback intensity, the external round-trip delay time, the angular frequency of the solitary laser, the phase delay during the external round-trip time t_d and the frequency chirp calculated according to [26].

The above equations may provide much understanding of the operation of semiconductor lasers if small-signal perturbation model is applied. This means taking a small fluctuation superimposed on a DC component for every state variable, which results in the very important expressions for the resonant frequency and the damping factor of a semiconductor laser (section 5.12.2 in [4]):

$$\omega_r = \sqrt{\frac{a}{\delta_c}\frac{c_0}{n_r}\frac{N_{k0}}{t_{ph}}} \tag{9}$$

$$\gamma = \frac{a N_{k0} c_0}{\delta_c n_r}\left(1 + \frac{\epsilon_c n_r}{a c_0 t_{ph}}\right) + \frac{1}{\tau_n} \tag{10}$$

In these expressions, a is the differential gain (defined as $a = dg/dn|_{n_0}$), δ_c is a term to take gain compression into account and is defined as $\delta_c = 1 + \epsilon_c N_{k0}$, in which ϵ_c gives the amount of compression. Finally, N_{k0} is the volume photon density and τ_n is the total carrier lifetime (for both radiative and nonradiative recombination phenomena).

From the above equations it is evident how an active material (as semiconductor quantum dots) with high differential gain leads to large modulation bandwith and high damping rate, thus favoring high-speed direct modulation.

On the other side of the story, however, the device performance under direct electrical current modulation is limited by lasing wavelength chirping. Essentially, this phenomenon may be understood as the broadening of the laser emission linewidth, i.e., as the instantaneous frequency shift from the steady state value esulting from the coupling between the real and immaginary parts of the complex refractive index of the active material in the laser cavity (Kramers-Kronig relation), $n_r = n' + jn''$ (see section 13.7 of [28]). Since the real part of n_r is related to the phase of the optical field in the cavity (it gives the propagation constant $\frac{2\pi n}{\lambda}$), whereas its immaginary part appears as a gain term in this time-space varying description for the optical field, such a coupling is sometimes referred to as phase-amplitude coupling. It may result from external electrical pumping of carriers into the cavity or even from spontaneous phase fluctuation (spontaneous emission, shot noise) inducing photon intensity modification.

In the literature, the phase-amplitude coupling is commonly quantified by the Linewidth Enhancement Factor (LEF, or α-factor) reported below, which clearly reveals the influence of a high differential gain on obtaining a low α-factor:

$$\alpha_H = \frac{\Delta n'}{\Delta n''} = -2k_0 \left(\frac{\partial n / \partial N}{\partial g / \partial N} \right) \tag{11}$$

In section 5.10.1 of [4], finally, the intrinsic laser linewidth is shown to be:

$$\Delta f = \frac{hf}{4\pi t_{ph}^2 P_{out}} \left(1 + \alpha_H^2\right), \tag{12}$$

where $\hbar$ is the reduced Planck constant, P_{out} is the optical power exiting the laser cavity and f is the laser radiation frequency.

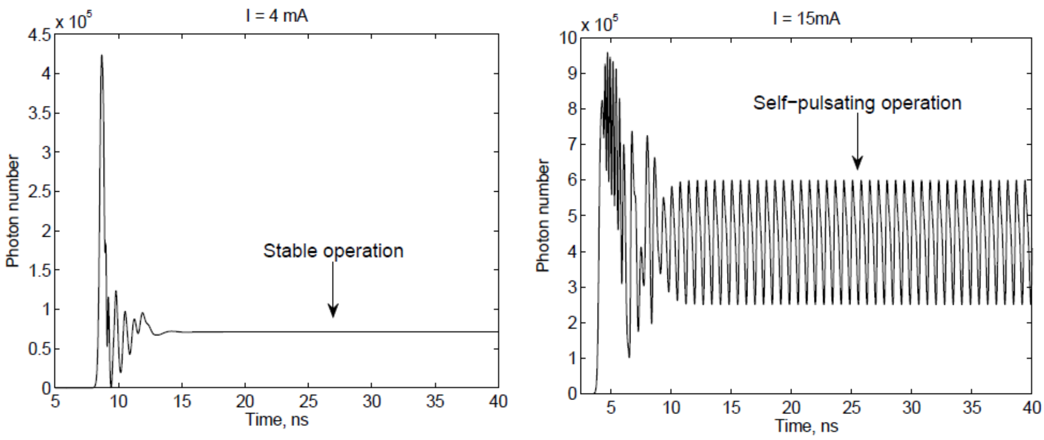
It is therefore expected that (ideal) quantum dot laser be robust to phase fluctuation, and as it is going to be discussed in the following sections, much work in the literature has dealt with estimating this parameter and studying how non-zero α-factor affects laser performance under optical feedback, especially if nonidealities are taken into account.

3. BASICS OF NONLINEAR SYSTEMS THEORY

The dynamical system of equations 1 - 8 constitute a set of 1^{st} order ordinary differential equations which couple the state variables $n_w, n_{ES2}, n_{ES1}, n_{GS}, n_s, n_d$... with no explicit time-dependence; it is, therefore, an autonomous system and, besides, it is nonlinear (recall the aforementioned stimulated emission terms, which depend on the square of the electrical field amplitude, $|E|^2$). Finally, since the coefficients multiplying the state variables do not have any spatial dependence, but implicitly depend on time (the gain, g, for instance, depend on the carrier number, which is a state variable), it is further classified as nonlinear variable-parameter system.

Dynamical system very often have solutions which can be an equilibrium point (or a steady-state time trace) or a cyclic curve in the phase-space (self-sustaining periodic oscillations), as illustrated in Figure 3 below.

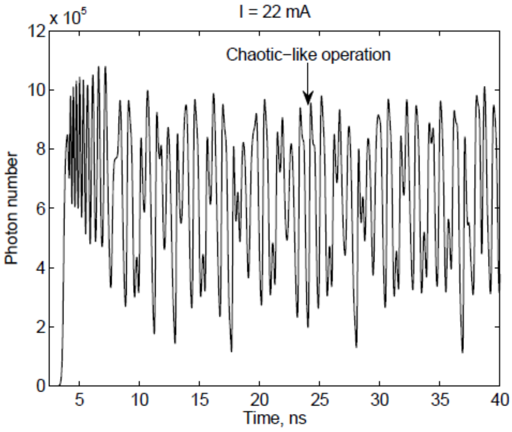
Figura 3: Temporal laser response for different current injection levels



In nonlinear systems, it can be found another very typical time trace which is characterized by an irregular behaviour like that of Figure 4.

This look apparently unpredictable for time function can not be associated to a random process because it comes from a known set of deterministic equations. Indeed, such an erratic-looking time trace is actually an indicator of possible chaotic behaviour. According to [29], "chaos is a sustained... (see page 17)". In [30], a very clear condition for chaos occurrence is

Figura 4: Temporal laser response for higher current injection level



given as: "only nonlinear dissipative system may experience chaotic behaviour". This last definition provides a useful way to proceed when qualitatively evaluating a system because the nonlinear nature of a given system is easy to check (by inspecting or by applying the superposition theorem), and the dissipative nature requires computing and checking the divergent of the velocity field as:

$$\vec{\nabla} \cdot \vec{f} = \frac{\partial f_1}{\partial x_1} + \frac{\partial f_2}{\partial x_2} + \cdots + \frac{\partial f_n}{\partial x_n} < 0 \tag{13}$$

The above equation reveals that in a dissipative system the n-dimensional volume shrinks over time. This means that a hipervolume in the vicinity of an initial condition $(N_{GS}(0), N_{ES1}(0), ...)$ shrinks in such a way that the system solution moves towards a limited region of the phase-space, which is called attractor. In a bidimensional system, for instance, this region can be a point in the n-dimensional space (zero-dimension attractor) or a limit-cycle (one-dimension attractor). In higher order systems, however, it is also possible to have trajectories defining a torus-like surface (two-dimension attractor) or, ultimately, the so-called strange attractor. In a strange attractor the trajectory never visits a given point in the space more than once.

Another characteristic of a strange attractor is that it can not be assigned an integer capacity dimension, but a fractionary number describes it instead. That is why it is common to refer to the fractal dimension of a strange attractor. Such a capacity dimension is written as

$$D_0 = \lim_{\epsilon \to 0} \frac{\log N(\epsilon)}{\log(1/\epsilon)} \tag{14}$$

The capacity dimension of an attractor mentioned so far measures the minimum number of coordinates necessary to localize every point of the attractor itself; it reveals the number of variables necessary for describing the asymptotic behavior of a given system. Attractors having $D_0 = 0$, for example, are characterized by fixed-point solutions, whereas $D_0 = 1$ or $D_0 = 2$ indicates the system evolves to a limit-cycle or a torus in phase space, respectively. Formally, this can be written as

Formally, according to [31] an attractor is defined as a closed invariant set $L \subset R^n$ for which there exists a neighborhood U of L such that

$$\forall x \in U \, and \, \forall t \geq 0 \Rightarrow \phi(x,t) \in U \, and \, \lim_{t \to \infty} \phi(x,t) \to L \tag{15}$$

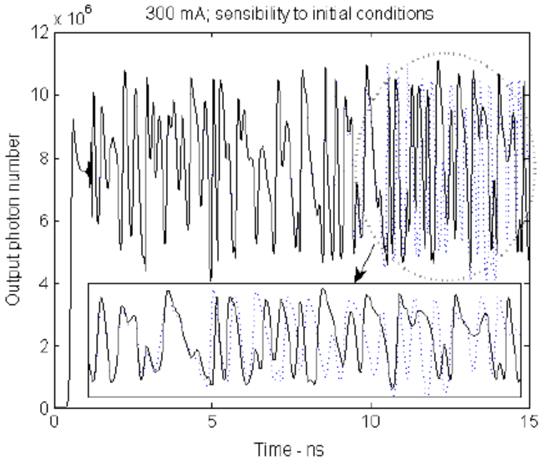
In words, an attractor is nothing else than a region L in phase space (and, hence, a collection of points in R^n) towards which trajectories starting in the vicinity U of L evolve over time. The above discussion is important because it links the asymptotic evolution of a given system and the existence of chaotic behaviour. Indeed, chaos is characterized by convergence towards a strange attractor in the phase-space.

Aside the unpredictable nature of the dynamic response, another very peculiar characteristic (and perhaps the main one) of a chaotic system is its sensitivity to initial conditions. This unique feature of chaotic operation can be explained as it follows: if a given chaotic system is subject to just slightly different values of starting conditions in two repetitions of a given experiment, then the trajectories in the phase space follow divergent paths in the two cases. Differently, if the system is in nonchaotic regime, then the two trajectories get close together or remain equidistant as time goes. Put in other words, according to [29]: "the slightest change in a variable's first value or the system's state at one time leads ultimately to very different evolutionary paths".

To illustrate the concept, in Figure 5 below presents the time traces obtained after a 300 mA switch-on driving in two different experiment where very close initial conditions are taken. The two time-series look overlap in the beginning, but then they deviate from t = 9.5 ns, showing completely different responses at longer time instants.

Such a long-term consequence of a tiny fluctuation in initial condition got especially relevant after Lyapunov work on a quantitative measure for this system' sensitivity, the Lyapunov exponents. What Lyapunov exponents do measure is the average rate of the exponential divergence of adjacent trajectories (those enclosed in a hypersphere with diameter d_0). As time evolves,

Figura 5: Laser response for two slightly different initial conditions.



this diameter may change according to [31]:

$$d(t) = d_0 e^{\lambda t} \tag{16}$$

From this equation, it is clear that for positive lambda the original hypersphere get bigger and bigger, revealing exponential divergence and, therefore, the existence of chaos.

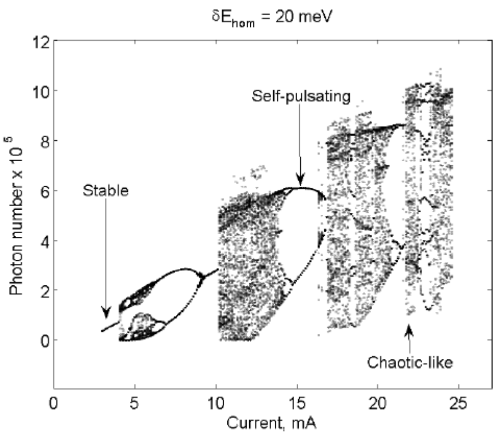
If you now consider that the divergence rate of the trajectory may differ from one direction of the n-dimensional space to another, then you can define the Lyapunov exponent associated to the j-th direction:

$$\lambda_j = \frac{1}{N} \sum_{n=1}^{N} ln \left(\frac{F_j^{(n)}(x_0 + \delta_0) - F_j^{(n)}(x_0)}{\delta_0} \right) \tag{17}$$

where n is the discrete-time index, δ_0 is the small distance between two close initial conditions and N is the size of the discrete time-trace vector, provided the transient is discarded. Therefore, the whole set $\Lambda = \{\lambda_j\}$ constitute the Lyapunov spectrum. From the calculated spectrum, it is enough to check the larger exponent; if $\lambda_{max} > 0$, then the system is chaotic and its trajectory in phase space will sooner or later reach a strange attractor.

As already pointed out formerly, the time-series traces presented in Figures 3 and 4 represent the different responses that a given nonlinear system may present when subject to different operating conditions (by operating conditions it is meant a scenario in which some system parameter is suitably changed); in the particular case of those figures, the driving electrical current was the system parameter assuming values equal to 4 mA, 15 mA and 22 mA, respectively. A very useful tool to illustrate such a qualitative dependence of the time-response of a system on a single parameter value is to depict the bifurcation diagram, like the one in Figure 6.

Figura 6: Bifurcation diagram of a quantum dot laser under optical feedback



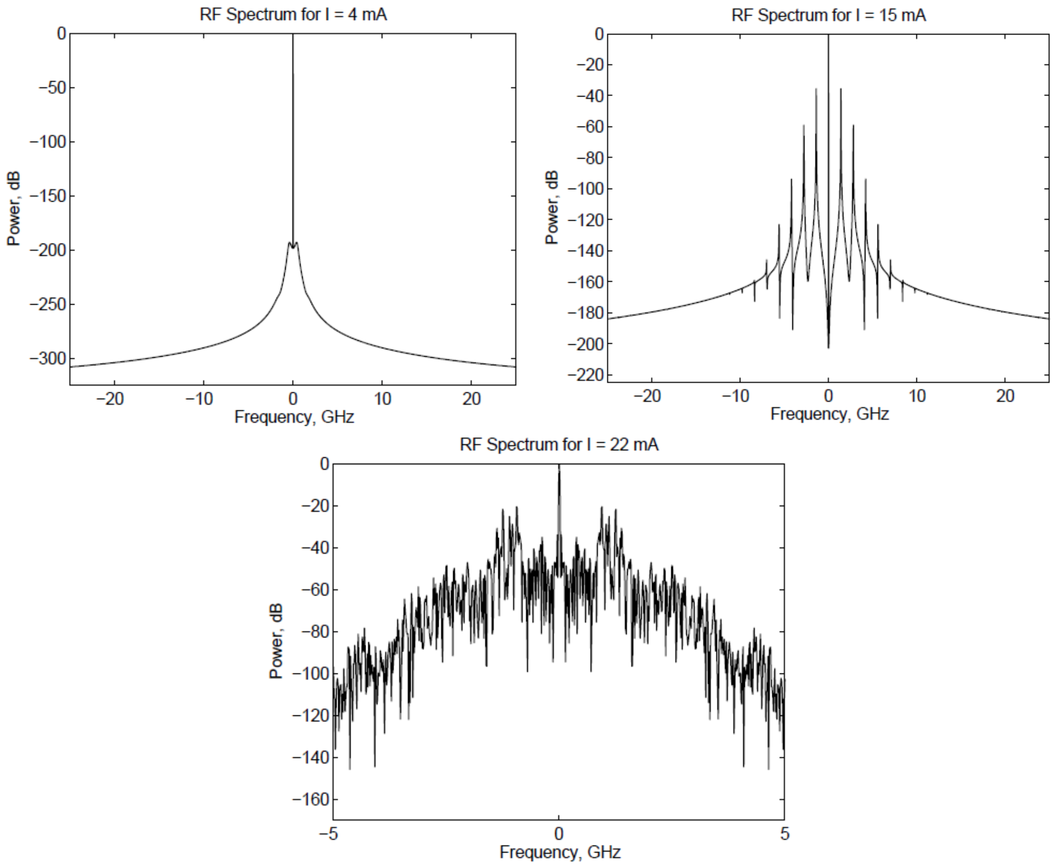
Formarly, the simplest definition of a bifurcation states that it is ”any modification of the asymptotic behaviour obtained as response to changing a parameter value” [32]. A bifurcation diagram shows, therefore, in the y-axis the system output after eventual switching-on transient (as time approaches infinity) and, in the x-axis the control parameter. It is very characteristic of bifurcation diagrams the existence of clear transitions between two regions in which qualitatively different solutions are observed;

a transition occurs whenever the control parameter changes (even slightly) around a critical value. In the particular case of Figure 6, I = 10 mA is a critical value after which the device enters a chaotic regime and I = 9 mA is a critical value for which a transition from oscillatory to constant response is observed.

By looking at the bifurcation diagram, therefore, one is able to better understand chaos in a particular system, or even identify patterns and anticipate occurrence of chaos; in applications of optical communications, chaotic laser sources are twofold: whereas in continuous-wave operation the experiment is designed to prevent the occurrence of chaos in light intensity because it means unacceptable uncertainty for the application, in chaotic communications, it is instead a requirement to have the laser source emitting like chaotic time-series. An interesting work on chaotic communications is discussed in [33].

At last, it is worth discussing a further way to view chaos, to realize its occurrence in a given dynamical system, namely the power spectrum. As the system evolves from a completely stable and stationary state like that of Figure 3 (left) to that of Figures 3 (right) and 4, what we observe in the frequency domain is a power spectrum changing from a single line centered at zero frequency (Figure 7 left), to a scenario with a finite number of spectral lines (each one denoting the existence of a periodic contribution), Figure 7 (right) and, ultimately, to a continuous spectrum with the energy spreading over a wide bandwidth, where peaks are not clearly seen anymore and the system entered the chaotic state, Figure 7 (bottom).

Figura 7: Power spectrum for different current injection levels



4. PREVIOUS AND RECENT RESULTS

Most of the interest in the performance of QD lasers under optical feedback started in 2003 with a short letter by [34]. They considered the problem of reducing the packaging cost of the laser by suppressing the optical isolator and studied the effects of that on the device performance. It was already known that external reflections could lead to performance degradation and the paper contributed to the community by finding that QD lasers were much insensitive to fed back light.

The same authors moved into the physics of the semiconductor material to further investigate the sensitivity in [22], this time focusing on the electron average capture rate. [1]

In what followed, the sensitivity to optical feedback appeared again in the work of [24]. Based on results of [34], which considered a long device for 1.3um operation with low α-factor, the authors investigated the dependence of such robustness to feedback on device length. In this study shorter lasers revealed to be more sensitive to feedback than longer ones, and explanation was given in terms of slower relaxation oscillations dynamics in long lasers. This trend was confirmed again in 2007 by means

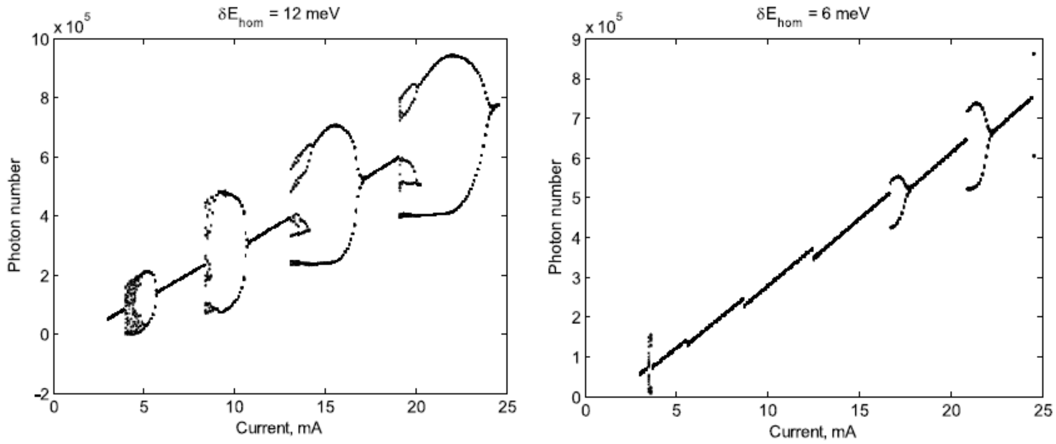
[1] The electron capture may be seen as a scattering phenomenon in which the carrier occupies a 0D energy state (full-spatial confinement) after a transition (according to Fermi Golden Rule) from a 2D or 3D (no confinement) energy state. For a review on this microscopic modeling, refer to Appendix 2 in [**?**]

of a time-delayed differential equations model, which was very useful to show the condition for dynamical instabilities an route to chaos in the power-time response [23].

By this time, two conference contributions brought to the community many new insights on the physical motivations for the chaotic behaviour of such devices. In the first one [35], a multi-populations model was introduced for simulating QD laser under optical feedback; the perfomance of the device was studied in terms of bifurcation diagrams and power spectrum for different electrical operating conditions. Furthermore, the fluctuation of laser linewidth was analyzed and different phase-space portraits revealed that a specific population of charge carriers in the device (namely, those occupying the wetting-layer energy states) were more responsible for the observed chirp. Also, in that work, phase-space portraits showed the various operating conditions leading to fixed-point, limit cycles as well as to strange attractors.

In the second work [36], authors provided another systematic study of sensitivity to feedback and, by means of the bifurcation diagrams of Figure 8 below they showed how the quantum dot material gives more stable devices as the homogeneous broadening of the optical gain approaches the bottom limit of 5 meV. Recalling from the literature that this parameter in quantum dot material increases with the injection level [37], one can conclude that at low levels of injection current QD laser operation is feedback-robust.

Figura 8: Bifurcation diagram as the homogeneous broadening approaches the bottom limit: from 12 meV (left) to 6 meV (right).



To complete the initial picture of this survey, it is important to comment about another work which expanded the fundamental knowledge on the α-factor of QD lasers. In [21], the parameter studied was the operating temperature, which revealed to be rather influential by making α to increase from 1.5 at 20^oC to 5.0 at 50^oC, inevitably leading to irregular power drop-outs, periodic pulsation and route to chaos. In that work, the experimental analysis was mainly based on power spectra like those of Figure 7.

From 2008 to 2012 a project in Europe, FASTDOT, made huge efforts for the development of high-performance QD laser sources. In the project, researchers were interested among other purposes, in the exploitation of chaotic QD lasers for wideband light emission. In [38], for example, they reported on the use of external cavity configuration (like that of Figure 2 and showed that emission spectrum around the GS transition raises up to 50% when -23dB optical feedback re-enters the laser cavity.

More interestingly, in the same report the authors studied the appearance of chaos even in the absence of any external mirror; they characterized and confirmed chaos occurrence from experimental time traces. Authors calculated the correlation dimension D_2 of the strange attractors [2] according to Grassberger and Procaccia approach, which is a topology-based method for evaluating the fractional dimension of strange attractors, and found many non-integer correlation dimension values in the interval $1 < D_2 < 7$ for many different scenarios of current injection.

As explained in section 3, another approach for characterizing the occurrence of chaos in QD lasers is the calculation of the spectrum of Lyapunov exponents, what has been considered in recent conferences papers. In [39], the influence of the laser cavity length on the sensitivity to optical feedback from an external mirror was revisited; calculations revealed that the dynamical system representing the operation of longer devices (1 mm length) does not present any positive Lyapunov exponent, whereas for shorter lasers (700 and 600 μm) there is one positive exponent along the 140^{th} direction of the phase space (that representing the electrical field intensity in the laser cavity). Additionally, it was also studied the influence of the external cavity length in the sensitivity to initial conditions for short laser devices; the conclusion was that the longer the external cavity the sooner adjacent trajectories start to deviate from each other.

In [27] the author moved his attention from design parameters (laser cavity, external cavity length) to physical parameters. The direct capture scattering phenomenon was included in the rate-equations model and its influence on the chaotic behaviour was studied, with hiperchaos appearing at different electrical driving scenarios. It was revealed that at low current injection levels, the largest positive Lyapunov exponent obtained when the direct capture phenomenon is present was almost the double that of when it was neglected. For higher injection levels, however, the largest positive exponent did not depend on the existence of the additional scattering channel. To explain that trend, we may use the results of [35] and recall that most contribution to lasing chirp comes from the wetting layer population; at high injection current levels, when it is fully populated, the existence of

[2]since D_2 is upper-limited by D_0, estimating D_2 is enough to give a lower-limit for D_0 and characterize chaos

the additional scattering channel is not much influential. Hence, the chirp (and, therefore, the tendency to unstabilities) does not differ much when the direct capture phenomenon is included. At lower injection current, instead, the wetting layer population is rather influenced by the additional drain of carriers to the quantum dot states and the lasing frequency fluctuates more, in turn. This finally leads to more unstable scenario, with faster divergence of adjacent phase-space trajectories, represented by higher positive Lyapunov exponents.

5. HIGHLIGHTS AND CONCLUSIONS

In this paper a survey of the findings and progress of the last 10 years of research in the field of nonlinear analysis of quantum dot laser was presented. From the first initial studies about the influence of the design issues on the stability of laser devices to the more recent explanations based on physics behind the operation of chaotic QD lasers, a number of papers have reported different approaches to characterize the occurrence of chaos in these devices. Among them, the power spectrum, the correlation dimension of the strange attractor, the bifurcation diagrams and the existence of positive Lyapunov exponents were preferred.

Nonetheless, all the papers and contributions here discussed were limited in scope, in the sense that an external cavity model was adopted as basic configuration for the modeling. This is fine for studying the effects of undesirable back-reflections, and it might be interesting for industrial experimentalists producing low-cost laser packages for optical communication.

Beyond this simplification, however, just recently a deeper bifurcation analysis on the instabilities of quantum dot laser has been published. In [40] the authors considered the problem of modeling not only the time delayed feedback of the laser light, but also the optical injection from a different laser source. From very interesting two-parameters bifurcation diagrams and based on a microscopic description of the carrier scattering phenomena, authors could conclude that most of the expected tolerance of QD lasers to optical perturbations is, to a large extent, due to independent dynamics of the material gain and refractive index changes, and predictions based on the α-factor may fail in certain scenarios.

These results point to new directions to be followed in near future research, with microscopically based balance equation model being a must.

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