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SUPPLEMENTARY MATERIAL DYNAMIC MULTI-OBJECTIVE EVOLUTIONARY ALGORITHMS: AN OVERVIEW

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1 DMOEAs design

Table 1 shows a summary of our coverage on the literature on DMOEAs structure. We can see that the Pareto-based DMOEAs are the most used alternative.

Table 1: MOEAs structures within the DMOEAs: Pareto-based (PB), Indicator-based (IB), or Decomposition-based (DB)

Type	DMOEAs
PB	[1–42]
IB	[43, 44]
DB	[1, 39, 45–61]

Table 2 shows a summary of the literature review of the variation operators used in DMOEAs. We can observe that the polynomial mutation and simulated binary crossover are the most commonly used operators.

Table 2: Types of Variation Operators based on those found in genetic algorithms (GA), Gaussian mutation and arithmetic crossover (GM-AC), differential evolution (DE), polynomial mutation and SBX crossover (PM-SBX), differential evolution and polynomial mutation (DE-PM), evolutionary operator choice (EOC), adapted operators (AO), RM-MEDA, IM-MOEA, and teaching-learning-based (TLB)

Traditional	DMOEAs
GA	[6, 12, 46]
GM-AC	[7, 16, 44, 45]
DE	[1, 17, 18, 30]
PM-SBX	[1, 5, 10, 11, 15, 19, 22–25, 28, 29, 31, 35, 36, 38, 39, 42, 50–52, 54, 55, 62]
DE-PM	[26, 42, 48, 49, 57–61]
EOC	[3, 44]
AO	[2, 9]
Learning-based	**DMOEAs**
RM-MEDA	[4, 20, 21, 27, 31–33, 37, 39]
IM-MOEA	[53]
TLB	[34, 56]

2 New DMOEAs Taxonomy

Table 3 shows a summary of the studies in each group of the new taxonomy. We can see that the prediction-based approaches are the most common.

3 Handling Constrained DMOPs

It is worth discussing the dynamic constrained multi-objective optimization problems (DCMOPs) which are characterized by dynamic constraints and/or the dynamic fitness functions (Azzouz et al. [68], Chen et al. [69]). To solve them, Li et al. [70] transform a constraint optimization problem (COP) into a corresponding dynamic constrained many-objective optimization

Table 3: DMOEA Taxonomy

Approaches	Methods
	Endogenous
MOEAs-Based	Variation Operators: [2, 8, 15, 18, 41, 46] Local Search: [22, 63]
Populations-Based	Functional decomposition: [7, 9] Data decomposition: [6] Hybrids methods: [64]
	Exogenous
Memory-Based	Explicit memory: [51, 57] Local-Search memory: [44] Hybrid memory: [11, 24]
Immigrant-Based	Uncorrelated Immigration schemes: [5] Correlated Immigration schemes: [5, 16, 17, 25, 37, 40] Hybrid Immigration schemes: [52]
Prediction-Based	Decision Space: [3, 4, 13, 14, 19–21, 27, 32, 33, 35, 36, 38, 39, 42, 48–50, 54, 56, 58–62, 65–67] Objective Space: [31, 34]
	Endogenous-Exogenous
	[26, 28, 30, 47, 53, 55]

problem (DCMaOP), by converting an m-constrained problem into an $m + 1$ DMOP and, then, using a dynamic constraint handling mechanism. Azzouz et al. [68] deal with DCMOPs simultaneously considering problem constraints and objective functions. The authors introduced a dynamic and self-adaptive penalty function to respond to changes in the fitness function and a feasibility driven strategy that is a repair mechanism triggered when the CRM detects a change. Jiao et al. [71] turn a constrained problem into an unconstrained one, taking the constraints as new objectives. Specifically, they introduced an algorithm for handling all constraints and objectives together. The algorithm converts a CMOP into a DMOP or a weakly constrained MOP and solves it. Chen et al. [69] proposed an algorithm in which the selection mechanism can consider feasible and infeasible solutions as potential parents or survivors, and the unfit, even though viable, solutions can be reconsidered after a change.

Very often, EAs to solve DCMOPs borrow a number of strategies used by constrained MOEAs ([72,73]) to handle constraints: penalty functions, special representations and operators, repair mechanisms, and separation of constraints and objectives. Chen et al. [69] argue for there to be three classes of strategies to deal with DCMOPs: giving the highest priority to the survival of feasible solutions, seeking a balance of feasibility and convergence by the proper mechanism, and repairing infeasible solutions. These may be taken as the main current approaches for handling DCMOPs by DMOEAs.

4 Dynamic Multi-Objective Problems

Table 4 summarizes the DMOPs covered in this survey when handled by DMOEAs. We can observe that FDA functions and dMOP benchmarks are the most used problems. Also, we notice that there is not a standard problem set to DMOEAs assess. Furthermore, although many real-world dynamic multi-objective optimization problems exist, very few DMOEAs have been used to solve them.

Table 4: Dynamic Multi-Objective Optimization Problems and the studies that use them.

DMOPs	**DMOEAs**
Moving Peaks Benchmark	[7, 13, 74]
FDA	[1–7, 9, 11–20, 22–24, 26–29, 32, 33, 37–39, 41–54, 56–61, 66, 67, 75, 76]
DSW	[1]
ZJZ	[4, 41, 48, 56, 59, 61]
$DTLZ_{Av}$	[28, 77]
dMOP	[12, 15, 18, 20–22, 26–29, 32, 33, 37–39, 41, 42, 48–51, 56, 58–60, 75, 76]
DIMP	[14, 16, 38, 42]
DMZDT	[11, 24, 44]
WYL	[11, 44]
HE	[38, 78]
ZJZ2 (F)	[20, 21, 26, 27, 32, 33, 37, 49, 58, 65–67]
UDF	[28, 41, 54, 56, 76, 79]

WYL2	[26]
MT	[49]
CEC2015	[31,34]
GTA	[53,66,67,80]
JY	[35,42,59–61,65–67,81]
DF	[36,39,42]
Fun	[66,67]
fun	[67]
DCMOPs	**DMOEAs**
DTF	[1]
DCTP	[25]
DMaOPs	**DMOEAs**
SJY	[29,82]
Real-World	**DMOEAs**
Control of combustion in a rubbish burner [45]	
Dynamic hydro-thermal power scheduling problem [5]	
Optimal force allocation for a combat simulation [46]	
Identification of good parameter sets for the machining gradient materials [10]	
Feature selection problem of the dynamic streaming data environments [40]	
Dynamic workflow scheduling in cloud computing [62]	

5 Performance indicators: definitions

Many measures used to evaluate MOEA performance have been adapted to assess DMOEAs. Helbig and Engelbrecht [83] classified these dynamic multi-objective performance measures into four groups: (1) accuracy performance indicators, (2) diversity performance indicators, (3) combined performance indicators, and (4) robustness performance indicators.

Accuracy performance indicators. These evaluate the algorithm's convergence. They can be applied to the distance between the approximate optimum front (POF^*) and the true one (POF'), or between the approximate optimal set (POS^*) and the true one (POS'); moreover, they are based on the optimal solution percentage/ratio found to belong to the POF/POS; thus, they measure the algorithm's accuracy in tracking the POF/POS. For a given $x_{1:N}$, the set of non-dominated solutions is of size N and can be found by the algorithm at the $t-th$ iteration. Table 5 presents the most popular indicators in this group.

Diversity performance indicators. Diversity indicators can measure either the distributivity of solutions in the POF^* or the extension of the resulting Pareto front. For a given $x_{1:N}$, the set of non-dominated solutions is of size N and is found by the algorithm at the $t-th$ iteration for a problem with m objective functions. Several indicators developed in this group are presented in Table 6.

Table 6: Diversity Performance Indicators

Indicator	Equation	Best
Spacing Metric of Deb (Δ) [89]	$\Delta = \frac{\sum_{k=1}^{m} d_k^e + \sum_{i=1}^{N} \lvert d_i - \bar{d}\rvert}{\sum_{k=1}^{m} d_k^e + N\bar{d}}$ where d_i is any distance between neighboring solutions, $\bar{d}$ is the distances average and d_k^e is the distance between the extreme solutions of POF^* and POF'. It is only used for bi-objective problems.	$\downarrow$
PL-Metric (PL) [1]	$PL := \frac{\sum_{f(x_i)\in POF'} \ln(\xi_{x_i})}{L_{POF'}}$ where $\xi = L(\gamma, f(x_i), f(x_{i+1})) + 1$ is each subsection between neighboring points arranged in the true Pareto front, adding 1 to ensure that new solutions increase the metric value. $L(\gamma, a, b) := \int_b^a \lvert\dot{\gamma}\rvert dt = \int_b^a \sqrt{\dot{\gamma}_1^2 + ... + \dot{\gamma}_m^2}$ is the path size between two solutions $[a, b]$. $\gamma(t) :\subseteq \Re \to \Re^m$ is a continuous parametric function.	$\uparrow$
Average Density (AD) [87]	$AD = \frac{1}{KT}\sum_{i=1}^{T}\sum_{j=1}^{K}\sqrt{\frac{1}{N-1}\sum_{l=1}^{N}(\bar{d_{ij}} - d_{ijl})^2}$ with $d_{ijl} = \min_{r\neq l, 1\leq r\leq N}\{\lVert\mathbf{x_l} - \mathbf{x_r}\rVert \mathbf{x_l}, \mathbf{x_r} \in X_{ij}\}$, and $\bar{d_{ij}} = \frac{1}{N}\sum_{i=1}^{N} d_{ijl}$	$\downarrow$
C-Metric (C) [9]	$C(A_i, B_i) = \frac{\lvert\{b\in B_i \lvert \exists a \in A_i : b \succeq a\}\rvert}{\lvert B\rvert}$ (A_i, B_i) are two sets of non-dominated solutions found by two algorithms. If $C(A_i, B_i) = 1$, B_i is weakly dominated by A_i. If $C(A_i, B_i) > C(B_i, A_i)$, algorithm A has better performance than B. The dominance operator is not symmetrical. To analyze how close POF^* to POF' is, the set A is formed by the solutions of POF', and the set B is formed from $POF*$.	$\uparrow$

Coverage Rate (Co) [87]	$Co(A,B) = \frac{1}{4TK^2}\sum_{i=1}^{T}\sum_{j=1}^{K}\sum_{r=1}^{K} C(A_{ij}, B_{ir})$ If $Co(A,B) > Co(B,A)$, algorithm A is better than B. To analyze how close POF^* to POF' is, the set A is formed by the solutions of POF', and the set B is formed from $POF*$.	$\uparrow$
U-Measure (U) [9]	$U_m(POF*) = d_{std} = \sqrt{\frac{1}{2mN-1}\sum_{g\in POF*}\sum_{r=1}^{2m}(d_r - d_{mean})^2}$ with $d_{mean} = \frac{1}{2mN}\sum_{g\in POF*}\sum_{r=1}^{m}(d_{2r-1} - d_{2r})$ where $r \in \{1,2,...,m\}$ is used to classify the $POF*$ into subsets $POF*_1^r; POF*_2^r; ...; POF*_k^r$ according to the r-th objective points value. $d_r = \frac{1}{2\lvert T-\{T_l^r, T_k^r\}\rvert}\sum_{g\in T-\{T_l^r,T_k^r\}}(d_{2r-1} - d_{2r})$, for any $g \in T_i^r \subset T$, $d_{2r-1} = d_{2r-1} + d_r$ se $i = 1$ and, $d_{2r} = d_{2r} + d_r$ if $i = k$. d_{2r-1} and d_{2r} are the distances of two neighbors of a point $g \in POF*$ from g.	$\downarrow$
γ Diversity Metric (γ) [43]	$\gamma = \frac{1}{NP}\sum_{j=1}^{NP}\frac{1}{L}\sum_{l=1}^{L}\left[-\sum_{k=1}^{S} P_{lk}\log(P_{lk})\right]$ where NP is the population size, L is the chromosome length, S is the genotype alleles cardinality, and P_{lk} is the k-th allele genotype rate in the l-th location.	$\uparrow$
Maximum Spread (MS) [90]	$MS(t) = \sqrt{\sum_{k=1}^{m}(\overline{POF_k^*(t)} - \underline{POF_k^*(t)})^2}$ where $\overline{POF_k^*(t)}$ and $\underline{POF_k^*(t)}$ are the maximum and minimum values of the k-th objective at POF^*.	$\uparrow$
Maximum Spread Adaptation (MS') [12]	$MS'(t) = \sqrt{\frac{1}{m}\sum_{k=1}^{m}\left[\frac{ms(t)}{\overline{POF_k'(t)} - \underline{POF_k'(t)}}\right]^2}$ $ms(t) = \min\left[\overline{POF_k^*(t)}, \overline{POF_k'(t)}\right] - \max\left[\underline{POF_k^*(t)}, \underline{POF_k'(t)}\right]$	$\uparrow$
Coverage Scope Measure (CS) [91]	$CS = \frac{1}{N}\sum_{i=1}^{N}\max\{\parallel f(\mathbf{x_i}) - f(\mathbf{x_j}) \parallel\}$ $\mathbf{x_i}, \mathbf{x_j} \in POF^*, i \geq 1$ and $j \leq N$	$\uparrow$
Spacing Metric of Schott (S or SP) [92]	$S = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(\bar{d} - d_i)^2}$ where $d_i = \min_{j=1,...,N}\{\sum_{k=1}^{m}\lvert f_{ki}(\mathbf{x}) - f_{kj}(\mathbf{x})\rvert\}$ and $\bar{d}$ is the average of all d_i values.	$\downarrow$

Combined performance indicators. These indicators can compare algorithms by simultaneously considering their convergence and diversity. Table 7 presents a representative set of indicators from this category.

Robustness performance indicators. Robustness measures evaluate how well the algorithm responds to environmental changes. Table 8 shows the metrics of this group.

Typically, the measures and metrics are calculated by considering all changes. An average value, ($\bar{\theta}$), is calculated as

$$\bar{\theta} = \frac{1}{num_{change}}\sum_{i=1}^{num_{change}}\theta_i, \tag{1}$$

where θ is the performance indicator used, num_{change} denotes the number of environmental changes, and θ_i is the θ value calculated before the $(i+1)$-th change occurs.

From the state-of-the-art review of performance indicators, we can see in Table 9 that the combined measures category is the most used; these evaluate DMOEAs according to their convergence and spreading. The inverted generational distance IGD and HV metrics are the most popular. The maximum spread MS and generational distance GD are used as diversity and accuracy measures, respectively. Notably, no set of standard performance indicators has yet been established for evaluating DMOEAs.

Table 5: Accuracy Performance Indicators

Indicator	Equation	Best
Convergence Performance Measure (e_x, e_f) [45]	$e_x = \frac{1}{N}\sum_{j=1}^{N}\min_{i=1:\lvert POF'\rvert}\left\lVert\frac{POF'_i(t)-POF^*_i(t)}{R(t)-U(t)}\right\rVert$ $e_f = \frac{1}{N}\sum_{j=1}^{N}\min_{i=1:\lvert POS'\rvert}\lVert POS'_i(t)-POS^*_i(t)\rVert$ where $U(t)$ is the utopia point [84], and $R(t)$ is the time-dependent nadir point [84]	↓
Based-Distance Performance Indicator ($D(P)$) [85]	$D(P) = \frac{1}{\lvert POF^*\rvert}\sum_{x\in POF^*}\lVert x-y(x)\rVert_2$ $y(x) = \arg\min_{y\in POF'}\lVert x-y\rVert_2$	↓
Success Ratio (SC_τ) [1]	$SC_\tau = \frac{\lvert\{x\mid f(x)\in POF'\}\rvert}{\lvert POF^*\rvert}$	↑
Generational Distance (GD) [86]	$GD_t = \sqrt{\frac{\sum_{i=1}^{N}d_i^2}{N}}$ where d_i are the minimum Euclidean distances of the function value of a solution to the Pareto front	↓
Convergent Ratio (CR) [87]	$CR = \frac{1}{KT}\sum_{i=1}^{T}\sum_{j=1}^{K-1}\frac{1}{K-j}\sum_{l=j+1}^{K}C(A_{ij},A_{il})$ where K is the number of times that an algorithm A runs on the i-th environment, with $1\le i\le T$, resulting in K sets of non-dominated solutions $\{A_{ij}\}_{j\ge1}^{K}$	↓
Variable Space Generational Distance (VD) [12]	$VD(t) = \frac{\sqrt{N\sum_{i=1}^{N}d_i^2}}{N}$ where d_i is the Euclidean distance between the i-th POS^* solution and the closest POS' member	↓
Convergence Metric (λ) [43]	$\lambda = \frac{1}{N}\sum_{j=1}^{N}\min\lVert POF^*-POF'\rVert$	↓
Average Hausdorff Distance (Δ_p) [88]	$\Delta_p(X,Y) = \max\{GD_p(X,Y), IGD_p(X,Y)\} = \max\left(\left(\frac{1}{N}\sum_{i=1}^{N}dist(x_i,Y)^p\right)^{\frac{1}{p}}, \left(\frac{1}{M}\sum_{i=1}^{M}dist(y_i,X)^p\right)^{\frac{1}{p}}\right)$ where $X = x_1,...,x_N$, $Y = y_1,...,y_M \subset \Re^m$ are non-empty finite sets, GD_p and IGD_p are the average of GD and IGD metrics, respectively, using the p-norm	↓

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Table 7: Combined Performance Indicators

Indicator	Equation	Best
Hypervolume (HV) [93]	$HV = Leb\bigcup_{x\in POF}[f_1(x), r_1]\times\cdots\times[f_k, r_k]$ $r = (r_1, r_2, \cdots, r_m)$ is a reference vector. It is mainly used when the POF' is unknown.	↑
Hypervolume Ratio (HVR) [94]	$HVR(t) = \frac{HV(POF^*(t))}{HV(POF'(t))}$	↑
Hypervolume Difference (HVD) [95]	$HVD = HV(POF') - HV(POF^*)$	↓
Inverted Generational Distance (IGD) [11]	$IGD_j = \frac{\sum_{i=1}^{N} d_i}{N}$ with $d_i = \min_{k=1}^{\|POF'\|}\sqrt{\sum_{j=1}^{m}\left(f_j^{*(i)} - f_j^{'(k)}\right)^2}$	↓
Improvement version of IGD ($IGD+$) [96]	$IGD_j = \frac{\sum_{i=1}^{N} d_i}{N}$ with $d_i = \min_{k=1}^{\|POF'\|}\sqrt{\sum_{j=1}^{m}\left(\max\left\{f_j^{*(i)} - f_j^{'(k)}, 0\right\}\right)^2}$	↓
Accuracy measure (acc) [97]	$acc(t) = \frac{HV_{max}(t)}{HV(POF^*(t))}$	↑
Alternative accuracy measure (acc_{alt}) [97]	$acc_{alt}(t) = \|HV(POF'(t)) - HV(POF^*(t))\|$	↑
Set Coverage Metric (η) [98]	$\eta = \frac{D(POF^*, POF')}{HV(POF')} + \frac{D(POF', POF^*)}{HV(POF')}$ where $D(POF^*, POF') = HV(POF + POF') - HV(POF')$ is D metric [90].	↓

Table 8: Robustness Performance Indicators

Indicator	Equation	Best
Stability ($stab$) [97]	$stab(t) = \max\{0, acc(t-1) - acc(t)\}$ It is used to quantify the effects of environmental changes on the accuracy of the algorithm (acc).	↓
Reactivity ($react$) [97]	$react(t,\epsilon) = \min\left\{t' - t \| t < t' < \tau_{max}, t' \in \aleph, \frac{acc(t')}{acc(t)} \geq (1-\epsilon)\right\}$ where τ_{max} is the maximum number of iterations. This calculates how long it takes for an algorithm to recover after an environment change.	↓

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Table 9: Performance Indicator and the studies that use them.

Indicators	Studies
	Accuracy Performance Measures
e_x e e_f	[45]
$D(P)$	[4, 19, 85]
SC	[1]
GD	[1, 18, 22, 27, 32, 35, 37, 50, 53, 60, 65, 75]
CR	[87]
VD	[12, 14, 16]
λ	[43]
Δ_p	[49]
	Diversity Performance Measures
Δ	[2]
$PL-metric$	[1]
AD	[87]
$C-metric$	[9, 50, 87]
Co	[87]
$U-measure$	[9, 13]
γ	[43]
MS	[12, 14, 16, 18, 24, 25, 28, 37, 39]
CS	[34, 91]
S/SP	[28, 29, 32, 35, 37, 39, 50, 56, 60, 62]
	Combined Performance Measures
HV	[6, 15, 25, 33, 34, 42, 44, 47, 58, 62, 77]
HVR	[5, 22, 24, 25, 36, 51]
HVD	[4, 28, 35, 56, 60]
IGD	[11, 17, 20–29, 31–35, 37, 39, 41, 42, 44, 48, 49, 52–54, 56, 58–61, 65, 67, 75]
$IGD+$	[34]
acc, acc_{alt}	[6]
η	[77]
	Robustness Performance Measures
$stab$	[6]
$react$	[6, 31, 34]

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