Learning and Nonlinear Models – Journal of the Brazilian Society on Computational Intelligence (SBIC), Vol. 20, Iss. 1, pp. 47-61, 2022
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LINEAR MODELS APPLIED TO MONTHLY SEASONAL STREAMFLOW SERIES PREDICTION

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Abstract- Linear models are widely used to perform time series forecasting. The Autoregressive models stand out due to their simplicity in the parameter adjustment based on a closed-form solution. The Autoregressive and Moving Average models (ARMA) and Infinite Impulse Response filters (IIR) are also suitable alternatives since they are recurrent structures. However, their adjustment is more complex due to the problem having no analytical solution. This investigation performs linear models to predict monthly seasonal streamflow series from Brazilian hydroelectric plants. The goal is to reach the best achievable performance addressing linear approaches. We propose the application of recurrent models, estimating their parameters via an immune algorithm. In order to compare the optimization procedures, the Least Mean Square (LMS) and Recursive Prediction Error (RPE) algorithms are utilized. In addition, the AR model and the Holt-Winters method were performed. The results showed that the insertion of feedback loops increases the quality of the responses. The ARMA models optimized by the immune algorithms achieved the best overall performance.

Keywords- Seasonal streamflow series forecasting, Box & Jenkins Models, IIR filters, immune algorithm.

1 Introduction

Energy planning in Brazil is highly dependent on accurate predictions of monthly seasonal streamflows, since approximately 60% of the electric power is generated by hydroelectric plants (EPE, 2020; Siqueira et al., 2018). One of the most utilized prediction strategies by the Brazilian Electric Sector is the linear approach from the Box & Jenkins methodology (Box et al., 2008; Luna and Ballini, 2011; CEPEL, 2018; Siqueira et al., 2020). These models are simpler than nonlinear models concerning the operation and mathematical tractability (Siqueira and Luna, 2019).

In this context, autoregressive models are highlighted since they are nonrecurrent structure and their coefficients can be calculated by a closed form-solution, which allowed an elevated computational efficiency (Haykin, 2001; Siqueira and Luna, 2019). These equations provide the model with a global optimum of the cost function based on the mean square error (MSE) (Box et al., 2008).

Notwithstanding, streamflow series have statistical behavior, such as seasonality and non-stationary, being arduous in their treatment and predictions. These characteristics occur because their formation process depends on the rainfall throughout the year in a vast country like Brazil (CEPEL, 2018; Siqueira et al. 2020).

In this sense, there is stillroom for developing good linear solutions since it can use a robust approach with simple mathematical treatment and good performance. In this way, the introduction of feedback loops in linear models is particularly relevant because they provide more information to perform the output response (Siqueira et al., 2019).

However, some attention is necessary when we are working on recurrent linear models (Shynk, 1989; Haykin, 2001): i) these models do not allow the achievement of a closed-form solution to find their optimum parameters; ii) as indicated in the previous item, it is necessary to search values interactively to the parameters of the model. Once exists recurrence, the use of a classic nonlinear optimization method based on derivative information, can lead to an unstable solution; iii) in recurrent models, the cost function based on the mean square error (MSE) may have local optima; iv) the achievement of the derivatives of the cost function based on the free parameters is relatively complex, because of the existence of local optima.

On the other hand, the use of recurrent models raises the prospect of obtaining linear solutions with high generality, or, in other words, ideally, we can extract the best performance possible to a linear approach, with the global optimization of their parameters, based on the MSE function (Siqueira and Luna, 2019).

Considering the importance of the streamflow series forecasting for a country with high dependence on hydropower, we propose using immune algorithms (IAs), a bioinspired population metaheuristic (Puchta et al., 2016), to calculate the free coefficients of the recurrent linear models to solve the problem. The IAs are viable candidates because they present desirable characteristics to overcome the difficulties above (Castro, 2006; Puchta et al., 2020), such as: i) these algorithms realize the search process interactively; ii) since the algorithms are population, the achievement of unstable solutions do not compromise the search process. Bad solutions tend to disappear during the optimization process; iii) IAs present a global search mechanism based on their implicit parallelism or, in other words, their capability to explore simultaneously several regions of the search space; iv) it is not necessary to manipulate the cost function. We use a version of the CLONALG algorithm (de Castro and Von Zuben, 2001) adapted to work with real parameters.

As a way of evaluating the proposed approach's performance, we consider two recurrent formulations: a) Autoregressive and Moving Average Models (ARMA), from the Box & Jenkins methodology (Box et al., 2008); b) infinite impulse response filters, which we will abbreviate as IIR filters, once this is the classical structure of a recurrent filter (Haykin, 2001). As means to analyze the optimization performance of IAs, we performed two algorithms based on the calculation of derivatives of the cost function based on the MSE: least mean square (LMS) to both recursive models, and recursive prediction error (RPE), to the IIR filter (Skynk, 1989). In addition, we performed the nonrecurrent Autoregressive model (AR) to estimate the improvement obtained by using feedback loop (Box et al., 2008). Besides, the Holt-Winters model, developed to seasonal time series, is applied to compare the results (Box et al., 2008). Our goal is to evaluate the advantages of applying recurrent models together with a bioinspired approach compared to classic models.

The rest of this article is organized as follows: in Section 2 are the methodologies of linear prediction; Section 3 describes the LMS and RPE algorithms and Section 4 the immune algorithms. Section 5 is about the monthly seasonal streamflow series, the case study, and the analysis of the results. Section 6 presents the conclusions and future works.

2 Linear Prediction and Modeling

2.1 Autoregressive models (AR)

The Autoregressive model (AR) is one of the most widespread methods for stationary time series prediction because of its simple mathematical tractability. The AR is a FIR filter without feedback loops (Siqueira and Luna, 2019). The mathematical formulation of an AR model is presented in Equation (1):

$$\tilde{x}_t = \phi_1 x_{t-h} + \phi_2 x_{t-h-1} + \cdots + \phi_p x_{t-h-p+1} + a_t, \tag{1}$$

where x_{t-h}, $i = 1, 2, \ldots, p$ are the lags of the observed series, $\tilde{x}_t$ the predicted value in time t, ϕ_i, $i = 1, 2, \ldots, p$ is the free parameters, and the term a_t is the random component (Box et al., 2008).

This definition is set to the direct prediction approach for h steps ahead (Sorjamaa et al., 1997). The AR is adjusted by the Yule-Walker equations (Box et al., 2008), depicted in Equation (2):

$$\boldsymbol{\Phi} = \boldsymbol{R}^{-1}\boldsymbol{r}, \tag{2}$$

where the matrix $\boldsymbol{R}$ is given by Equation (3):

$$\boldsymbol{R} = \begin{bmatrix} r_0 & r_1 & \cdots & r_{p-1} \\ r_1 & r_0 & \cdots & r_{p-2} \\ \vdots & \vdots & \ddots & \vdots \\ r_{p-1} & r_{p-2} & \cdots & r_0 \end{bmatrix}, \tag{3}$$

and the vector **r** by Equation (4):

$$\boldsymbol{r} = \begin{bmatrix} r_h \\ r_{h+1} \\ \vdots \\ r_{h+p} \end{bmatrix}, \tag{4}$$

being r_i the estimate of the correlations of the inputs (Box et al., 2008).

The term h refers to the number of steps ahead that we intend to predict considering the direct approach (Siqueira et al., 2014), p is the order of the AR model and $\boldsymbol{\Phi} = [\phi_1, \phi_2, \ldots, \phi_p]$ is the vector of parameters. Equation (2) is the solution of the Yule-Walker Equations, which leads to a condition of minimum MSE, unique to each problem (Box et al., 2008).

2.2 Autoregressive and Moving Average Models (ARMA)

The Autoregressive and Moving Average models (ARMA) can be understood as a mix between the AR and a moving average (MA) models. While the AR combines the delays of an input signal, the MA combines random shocks a_t and its delays. A MA model with q order can be described by Equation (5):

$$\tilde{x}_t = -\theta_1 a_{t-h} - \theta_2 a_{t-h-1} - \cdots - \theta_q a_{t-h-q+1} + a_t, \tag{5}$$

where θ_j, $j = 1, 2, \ldots, q$, are the adjustable parameters.

Equation (5) is set to the direct prediction approach for h steps ahead. The MA model can be seen as an all-pole IIR filter, and unlike the FIR filters (Siqueira and Luna, 2019), there is no closed-form solution to calculate its free parameters because it is necessary to solve a nonlinear system (Haykin, 2001).

The ARMA model may contain adjustable zeros and poles, which turns this model more embracing than the others. The ARMA(p,q) for forecasting is described by Equation (6):

$$\hat{x}_t = \phi_1 x_{t-h} + \cdots + \phi_p x_{t-h-p+1} - \theta_1 a_{t-h} - \cdots - \theta_q a_{t-h-q+1} + a_t, \tag{6}$$

where ϕ_i, $i = 1, 2, \ldots, p$ and θ_j, $j = 1, 2, \ldots, q$, are its free parameters (Box et al., 2008).

A second formulation is proposed in this work, with the feedback of the error of the last iteration, to any number of steps ahead, according to Equation (7) :

$$\hat{x}_t = \phi_1 x_{t-h} + \phi_2 x_{t-h-1} + \cdots + \phi_p x_{t-h-p+1} - \theta_1 a_{t-1} - \cdots - \theta_q a_{t-q} + a_t. \tag{7}$$

Observe that to one step ahead prediction, Equations (6) and (7) are the same. The random shocks a_t are equivalent to the errors e_t defined in Equation (2), which feedback to the model (Box et al., 2008).

Analogously to MA, the optimization of the free parameters of the ARMA model requires solving a nonlinear system. However, in an ideal case, if the choice of the parameters and the order of the model is suitable, we have an optimum linear predictor (Box et al., 2008).

2.3 IIR FILTERS

Linear structures IIR can be built from a different perspective of the ARMA models, with the feedback of the output response of the model. This model has the following Equation (8) (Haykin, 2001):

$$\hat{x}_t = c_1 x_{t-h} + \cdots + c_p x_{t-h-p+1} - b_1 \hat{x}_{t-h} - \cdots - b_q \hat{x}_{t-h-q+1}. \tag{8}$$

in which c_i, $I = 1, 2, \ldots, p$ are the free parameters which weigh the feedforward inputs and b_j, $j = 1, 2, \ldots, q$ the weights of the feedback.

Like the ARMA models, the IIR filters have adjustable zeros and poles in their transfer function (Haykin, 2001).

2.4 HOLT-WINTERS MODELS

Time series may have complex behavior like seasonality. In this case, their prediction can be made by smoothing models. In this category are the Holt-Winters model (Morettin and Toloi, 2018), often used to series with this component.

The smoothing equations are based on three basic components: a) The level, which is stationary (without seasonality or trend) but may have some random perturbation; b) The trend component; c) The seasonality component.

The method has two variants: multiplicative and additive. We adopt the second proposal. It is important to observe that the streamflow series do not present trend (Box et al., 2008). Therefore, the trend component is considered equals to zero. The expression of the model to a horizon of h steps ahead is given by Equation (9):

$$\hat{x}_{t+h} = \bar{R}_t S_{t+h-L} + a_t, \tag{9}$$

where $\bar{R}_t$ is the level factor, S_t, the seasonal factor, a_t is the aleatory component, and L is the size of the seasonality. In our case, $L = 12$, or one year.

The smoothing equations are according to Equation (10) and Equation (11):

$$\bar{R}_t = A\left(\frac{x_t}{S_{t-L}}\right) + (1-A)(\bar{R}_{t-1}), \qquad t = L+1, \ldots, N \tag{10}$$

$$S_t = D\left(\frac{x_t}{\bar{R}_t}\right) + (1-D)(S_{t-L}), \qquad t = L+1, \ldots, N \tag{11}$$

where A, $0 \leq A$, and D, $D \leq 1$, are the smoothing constants which have to be optimized, and N the number of observations.

Another important stage in the application of this method is the initialization of the factors, based on the first L samples of the series, or one seasonal period of the historic data. For this purpose, we use Equations (12) and (13):

$$\bar{R}_t = \frac{1}{L}\sum_1^L x_t, \qquad t = 1, \ldots, L \tag{12}$$

$$S_t = \frac{x_t}{\bar{R}_t}, \qquad t = 1, \ldots, L \tag{13}$$

In the next section, the models and methods presented are tested in the monthly seasonal streamflow series forecasting.

3 Optimization Algorithms based on Estimates of the Gradient Vector

3.1 LMS Algorithm

A widespread algorithm in the literature to optimize linear filters is the least mean square (LMS) (Widrow and Hoff, 1960). This method can be used to calculate the parameters of IIR filters and ARMA models. The proof of the deduction is based on a stochastic approximation of the gradient vector, which allows search estimates of the Wiener solution with a low computational cost (Haykin, 2001).

The LMS minimizes the MSE through a stochastic search, reducing the cost function $\mathbf{J}_\mathbf{w}$ interactively based on the error produced at each interaction, or the instant error. Consider $\mathbf{J}$ a stochastic variant of $\mathbf{J}_\mathbf{w}$, endowed of a scalar factor equals to 1/2 for simplicity, which depends on a vector of parameters of the model $\mathbf{w} = [c_1, c_2, \ldots, c_p, b_1, b_2, \ldots, b_q]$, as in Equation (14):

$$\boldsymbol{J} = \frac{1}{2}\boldsymbol{e}_t^{\,2}, \tag{14}$$

where $\mathbf{e}_t$ is the error measured in the instant t. It is possible to calculate the derivative of $\mathbf{J}$ with respect to $w(\nabla_w \boldsymbol{J})$, to find the stochastic gradient according to an equation error model (Shynk, 1989) using Equation (15):

$$\nabla_w \boldsymbol{J} = -\boldsymbol{e}_t \boldsymbol{x}_t, \tag{15}$$

where x_t is not dependent of w_t. Lastly, using the classical form of the gradient method to minimization (Haykin, 2001) we have Equation (16) to update the weights:

$$\boldsymbol{w}_{t+1} = \boldsymbol{w}_t - \gamma \nabla_w \boldsymbol{J}, \tag{16}$$

Combining Equations (15) and (16), the expression of update of the coefficients is given by Equation (17):

$$\boldsymbol{w}_{t+1} = \boldsymbol{w}_t + \gamma \boldsymbol{e}_t \boldsymbol{x}_t, \tag{17}$$

where γ is the adjust step, and the vector $\mathbf{x}_t$ is composed of all the inputs, feedforward, and feedback. However, disregarding the influence of parameters on the feedback, the method can converge to a suboptimal solution, in terms of MSE (Shynk, 1989).

3.2 RPE Algorithm

Another method that utilizes information from the cost function **J**, instant estimate of the square error, is the Recursive Prediction Error algorithm (RPE). Its application here is restricted to the IIR filters. According to Shynk (1989), its expression has a simplification and is similar to the LMS algorithm, but the derivative calculation with respect to $w(\nabla_w \hat{\boldsymbol{x}})$ is different. In general form, it is possible to write the gradient vector as in Equation (18):

$$\nabla_w \boldsymbol{J} = -\gamma \boldsymbol{e}_t \nabla_w \hat{\boldsymbol{x}}_t, \tag{18}$$

where the gradient is the vector in relation of the weights $\mathbf{w} = [c_1, c_2, \ldots, c_p, b_1, b_2, \ldots, b_q]$ is given by Equation (19):

$$\nabla_w \hat{\boldsymbol{x}}_t = \left[\frac{\partial \hat{x}_t}{\partial c_1}, \ldots, \frac{\partial \hat{x}_t}{\partial c_p}, \frac{\partial \hat{x}_t}{\partial b_1}, \ldots, \frac{\partial \hat{x}_t}{\partial b_q}\right], \tag{19}$$

However, $\nabla_w \hat{\boldsymbol{x}}_t = \boldsymbol{x}_t$ is a simplification since the values of the gradient are dependent on the past values $\hat{\boldsymbol{x}}_t$. If we suppose that the values of the weights change very slowly, or in other words, $\mathbf{w}_t \approx \mathbf{w}_{t-1} \approx \ldots \approx \mathbf{w}_{t-N+1}$, when the step γ is small, it is possible to obtain recursive derivatives to the calculation of $\nabla_w \hat{\boldsymbol{x}}_t$ (Shynk, 1989). For this, it is necessary to store the past values of the series. It was proposed an additional step to simplify the calculation of derivatives, described in Equation (20):

$$\frac{\partial \hat{x}_t}{\partial c_t} \approx x^f_{t-k_1}, \qquad \text{and} \qquad \frac{\partial \hat{x}_t}{\partial b_t} \approx \hat{x}^f_{t-k_2}, \tag{20}$$

where $k_1 = 1, 2, \ldots, p-1$ and $k_2 = 0, 1, \ldots, q-1$, respectively.

In practice, an intermediate stage is added to the algorithm with the insertion of new vectors $\boldsymbol{x}^f$ and $\hat{\boldsymbol{x}}^f$, which are calculated instantly based on the original inputs and outputs. Therefore, each component of the gradient is simply a delayed version of the initial conditions. These vectors compose the expression of the update of the coefficients. The RPE algorithm is presented in Algorithm #1, which follows the original version from Shynk (1989):

Algorithm #1 - RPE algorithm

Definitions:

$$\boldsymbol{w} = [\boldsymbol{c_1}, \ldots, \boldsymbol{c_p}, \boldsymbol{b_1}, \ldots, \boldsymbol{b_q}]^{\boldsymbol{T}}$$
$$\boldsymbol{v} = [\hat{x}_{t-1}, \ldots \hat{x}_{t-q+1}, x_t, \ldots, x_{t-p+1}]^T$$
$$\boldsymbol{v}^f = [\hat{x}^f_{t-1}, \ldots \hat{x}^f_{t-q+1}, x^f_t, \ldots, x^f_{t-p+1}]^T$$

Initialization:

$$\boldsymbol{x}_{t-p} = \hat{\boldsymbol{x}}_{t-q} = \boldsymbol{x}^f_{t-p} = \hat{\boldsymbol{x}}^f_{t-q} = 0$$

For each new input x_t, d_t, do:

$$\boldsymbol{x}^f_t = \boldsymbol{x}_t + \sum_{i=1}^{p-1} c_i\, \boldsymbol{x}^f_{t-i}$$
$$\hat{\boldsymbol{x}}_t = \boldsymbol{w}^T_t \boldsymbol{v}_t$$
$$\hat{\boldsymbol{x}}^f_t = \boldsymbol{x}_t + \sum_{i=1}^{p-1} c_i\, \hat{\boldsymbol{x}}^f_{t-i}$$
$$\boldsymbol{e}_t = \boldsymbol{d}_t - \hat{\boldsymbol{x}}_t$$

Update of the coefficients:

$$\boldsymbol{w_{t+1}} = \boldsymbol{w_t} + \boldsymbol{\gamma v^f_t e_t}$$

where γ is the optimization step.

4 IMMUNE ALGORITHM

Immune algorithms or artificial immune systems (IAs), are bioinspired methods for search and optimization based on the defense mechanism against antigens of superior organisms (Castro, 2006).

The primary motivation to use this technique to optimize linear structures comes from the complex characteristics of the resulting optimization problem. The application of IAs were successfully in works related to signal processing and streamflow prediction compared to other methods as genetic algorithms (Attux et al., 2003).

In this work, we used a version of CLONALG algorithm (Castro and Von Zuben, 2002) with real codification. The algorithm's *modus operandi* is inspired in the process of recognizing antigens, where the solutions are vectors of real numbers (in the case treated here) and are equivalent to the structure of an antibody. The affinity between antigen and antibody is measured through the cost function named fitness, as usual in evolutionary computation.

The basic idea that rules this algorithm is the clonal selection principle. According to this principle, at the moment of recognizing an antigen, the defense cells create copies from themselves, and these clones are subject to mutations proportionally to the affinity between antibody and antigen (Castro and Timmis, 2002).

To use IAs is necessary to set some elements that are described below. The first is the fitness function, which is defined in Equation (21):

$$\boldsymbol{J_{fit}} = \frac{1}{(1+\hat{\boldsymbol{J}}_{\boldsymbol{w}})}, \qquad (21)$$

where $\hat{\mathbf{J}}_{\mathbf{w}}$ is the estimate of the cost function based on the mean square error.

The mutations associated with the clonal selection principle are Gaussian disturbances added to the real values of the individuals. Tin order to improve the potential of global search, the algorithm realizes a periodic insertion of new individuals that are randomly generated. The binomial cloning/mutation provides a local search mechanism. Algorithm #2 presents the immune algorithm used:

Algorithm #2 - CLONALG algorithm

Initialization
Choose the parameters of the algorithm and randomly initialize the antibodies of the population.

Interactive process
While the maximum number of interactions is not reached, do:

1. Calculate the fitness of all individuals;
2. Each N_{it} iterations, include N_{ind} solutions generated randomly and substitute for the N_{ind} antibodies with lower fitness;
3. Produce Nc copies of each antibody;
4. Apply a mutation antibody unchanged. The mutation is proportional to the cost and follows these two equations:

$$c' = c + \alpha N(0,1),$$
$$\alpha = \left(\frac{1}{\beta}\right) exp(-f),$$

where β is a regulation parameter of the mutation amplitude, N(0,1) describes a random value generated by a Gaussian distribution with zero mean and variance equals one, *exp* is the exponential function, and *f* is the value of the fitness of the clone *c*.

5. Determine the cost of the new individuals and save each group just the best solution.

5 Computer Simulations and Discussion

5.1 Case Study and Experimental Protocol

The stage of the computational simulations was performed with the historical series from three Brazilian hydroelectric plants.

- *Furnas*, located in Rio Grande river, with test set defined from 1952 to 1956 (dry) and from 1972 to 1976 (median);
- *Sobradinho*, in the border of the states of Bahia and Pernambuco, with the test set from 1981 to 1985 (wet);
- *Emborcação*, on the west of the state of Minas Gerais, from 1972 to 1976 (medium).

These test sets are usual in this kind of study and present a distinct hydrological behavior. It is possible to observe that the period from 1981 to 1985 presented an abnormally high volume of water caused by constant rainfall, inversely to 52/56, which was dry. In the period between 1972 and 1976, the mean is close to the historical mean of each plant. This variation in the hydrological behavior of the test sets allows evaluating the performance of the proposed predictors more broadly.

Each selected period has five years or 60 monthly observations. The Brazilian data of streamflow series are available on the website of Electric System National Operator (ONS, 2020). A brief descriptive analysis regarding their means and standard deviations are summarized in Table 1:

Table 1: Average and Standard Deviation for all series.

Series	Mean	Standard Deviation
Furnas	926.61	613.16
Sobradinho	2.66×10^3	1.95×10^3
Emborcação	486.07	362.80

It is notorious that *Sobradinho* plant has a historical mean twice and a half higher than *Furnas* plant, and *Furnas* is approximately twice compared to *Emborcação*. In Table 2 are presented the mean and standard deviation of each test period.

Table 2: Average and Standard Deviation for test sets.

Test Period	Mean	Standard Deviation
Furnas 52/56	656.41	409.09
Furnas 72/76	882.63	445.05
Sobradinho 81/85	3.47×10^3	2.22×10^3
Emborcação 72/76	433.01	273.42

Streamflow series related to hydroelectric plants present a characteristic seasonal component. It occurs because the volume of water is driven by rainfall throughout the year (CEPEL, 2018). Therefore, before applying the forecasting models, it is necessary to remove the seasonal component. This procedure is performed by using the monthly deseasonalization process, which is detailed in Equation (22) (Siqueira et al., 2012a; 2012b):

$$z_{i,m}\ \frac{x_{i,m}-\hat{\mu}_m}{\hat{\sigma}_m}, \qquad (22)$$

in which x is the original series that is transformed in a new stationary series z, $\hat{\mu}_m$ is the historical mean of the month $m = 1, 2, 3, \ldots, 12$, $\hat{\sigma}_m$ is the monthly standard deviation, m is the current month of the i-th year, $i = 1, 2, \ldots, N_y$. These mean and standard deviation are given by Equations (23) and (24) (Box et al., 2008; Siqueira et al., 2012c):

$$\hat{\mu}_m = \frac{1}{N_y}\sum_{i=1}^{N_y} x_{i,m}, \qquad (23)$$

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$$\hat{\sigma}_m = \sqrt{\frac{1}{N_y}\sum_{i=1}^{N_y}\left(x_{i,m} - \hat{\mu}_m\right)^2}, \tag{24}$$

where N_y is the number of samples from month *m*.

Initially, the series considering all subsets (training, validation, and test) pass through the deseasonalization process described in Equation (22). Next, the optimization algorithms to the models AR (Yule-Walker equations), ARMA (LMS), and IIR filters (LMS and RPE) were performed to all selected series to estimate their coefficients. After to ARMA and IIR models, the immune algorithm was implemented.

The parameters of the IAs were chosen by preliminary evaluations and set according to Table 3.

Table 3: Parameters of IAs to ARMA and IIR filter.

Parameter	Value
Number of antibodies (N_{ind})	10
Number of clones (*Nc*)	5
Changing amplitude control (β)	50
Number of reintegrated individuals	3
Periodicity of reintegration (N_{it})	20
Number of iterations	1,000

The addressed forecasting horizons were h = 1, 3, 6, and 12 steps ahead. The Friedman's test was applied to all predictions in the test sets, in real space, to analyze if the results were significantly different (Luna and Ballini, 2011). Lastly, the Holt-Winters model was utilized. The smoothing coefficients were calculated by an exhaustive search, with step equals 0.01.

The order of the models was defined by the analysis of the variance of the residuals (Box et al., 2008) in view of the principle of parsimony. It was selected a second-order AR model. After, studying the influence of the different forms of recursion, we choose just one feedback to ARMA models and IIR filters.

The considered metrics to analyze the errors were mean square error (MSE), mean absolute error (MAE), and mean absolute percent error (MAPE), exposed in Equations (25), (26) and (27) respectively:

$$MSE = \frac{1}{N_s}\sum_{t=1}^{N_s}(x_t - \hat{x}_t)^2, \tag{25}$$

$$MAE = \frac{1}{N_s}\sum_{t=1}^{Ns}|x_t - \hat{x}_t|, \tag{26}$$

$$MAPE = 100\frac{1}{N_s}\sum_{1}^{N_s}\left|\frac{x_t - \hat{x}_t}{x_t}\right|, \tag{27}$$

where x_t is the desired response,$\hat{x}_t$ is the predicted value, and N_s the number of data.

5.2 Results

Hereafter, in the Tables 4-7, are presented the computational results to the test sets. These results are the average of 30 simulations. The values in bold highlight the best performance achieved in each scenario. The acronym HW refers to the result achieved by Holt-Winters model, AR to the autoregressive model, ARMA1 to the model proposed in (6) and ARMA2 presented in (7), and IIR is the IIR filter. LMS, RPE, and IA correspond to the optimization methods. The term "deseas" means the error in the deseasonalized space or after the data pass through the transformation presented in Equation (25). We do not present the MAPE in the deseasonalized domain because it is necessary to divide the subtraction by a small number.

5.3 Analysis and Discussion

Initially, we applied Friedman's test to evaluate if the results were significantly distinct. The *p*-values achieved were below 0.05, which proves the hypothesis that the results are significantly different (Luna and Ballini, 2011).

The error metrics adopted are very usual in this kind of study. However, we observe that in the 16 best results found based on the MSE on the real domain, just in 11 the model showed the best MAE too. Similar behavior can be observed regarding the MAPE. To compare the results, we adopted as the best results those in which the model presents the lowest MSE, once the cost function is based on this metric (Siqueira et al., 2014; Siqueira et al., 2018).

Table 4: Results for all streamflow series considering one step ahead (h = 1).

	Model	MSE real ($\times 10^4$)	MAE real	MAPE real	MSE deseas	MAE deseas
FURNAS 52/56	HW	11.8920	206.3132	24.84	-	-
	AR	5.2963	153.8511	19.40	0.3084	0.4528
	ARMA+LMS1	**5.2434**	**152.2564**	**19.11**	**0.3031**	**0.4422**
	ARMA+IA 1	6.0848	162.3638	21.80	0.3401	0.4752
	IIR+RPE	5.4851	157.7098	20.14	0.3121	0.4591
	IIR+LMS	5.2808	153.6556	19.46	0.3053	0.4460
	IIR+ IA	5.6802	159.1192	21.37	0.3478	0.4740
FURNAS 72/76	HW	12.9324	270.8675	30.65	-	-
	AR	**4.3971**	**159.6183**	**17.33**	**0.3878**	**0.5385**
	ARMA+LMS1	4.5550	160.5871	17.36	0.3989	0.5427
	ARMA+ IA 1	4.4326	160.4042	17.63	0.4028	0.5503
	IIR+RPE	4.5894	161.6452	17.50	0.4077	0.5489
	IIR+LMS	4.5227	160.7465	17.39	0.3998	0.5441
	IIR+ IA	4.5753	160.8217	17.43	0.4061	0.5468
SOBRADINHO	HW	46.8939	1326.5386	30.27	-	-
	AR	11.7045	633.7137	14.53	0.3597	0.4383
	ARMA+LMS1	**11.1777**	**618.1714**	**14.26**	**0.3513**	0.4328
	ARMA+ IA 1	11.2139	618.1087	14.26	0.3525	**0.4327**
	IIR+RPE	12.1175	649.7864	14.73	0.3613	0.4442
	IIR+LMS	11.5907	631.5168	14.43	0.3555	0.4366
	IIR+ IA	11.4886	636.7802	14.64	0.3561	0.4433
EMBORCAÇÃO	HW	4.6990	135.1774	32.00	-	-
	AR	**2.8585**	**95.7394**	**18.91**	**0.5592**	**0.4987**
	ARMA+LMS1	2.8911	96.6709	19.12	0.5645	0.5021
	ARMA+IA 1	3.4639	103.1276	19.59	0.6161	0.5266
	IIR+RPE	2.8713	96.6585	19.13	0.5627	0.5026
	IIR+LMS	2.8724	96.7253	19.15	0.5630	0.5029
	IIR+ IA	2.8797	97.1362	20.06	0.5634	0.5082

It is possible to observe there is no perfect correspondence between the best MSE in the real and deseasonalized domain. This phenomenon may be related to the fact that the deseasonalization procedure commonly adopted in streamflow series forecasting considers all the samples of the series with the same importance (weight). Hence, certain periods with high variance exert a pronounced influence in the monthly mean and standard deviation, used in such a process. This undesirable effect could be mitigated by introducing penalties to the atypical periods of the streamflow series (Siqueira et al., 2014). In the test set, in 13 cases, there is a correspondence between the spaces.

The analysis of the results in the deseasonalized domain showed that for the 16 possible results in 14 the best results were for the ARMA model. The immune algorithm presented the best results in 8 cases and the LMS in 6. In just 2 cases, the AR model reached the best results, which is not surprising to h = 1, once the Yule-Walker equations guarantee global optimality in the MSE sense. The Holt-Winters model to the test sets could not overcome the performance of the other methods.

The general analysis based on the best MSE in real domain allows some critical considerations. The ARMA model achieved 13 of 16 best performances while AR 2, and the IIR filter 1. Furthermore, just comparing AR and IIR filter, the filter achieved the best results in 13 cases. It is clear that the insertion of feedback is an advantage, even losing the guarantee of the global minimum optimum, especially to horizons higher than 1 step ahead.

Table 5: Results for all streamflow series and three steps ahead (h = 3).

	Model	MSE real ($\times 10^4$)	MAE real	MAPE real	MSE deseas	MAE deseas
FURNAS 52/56	HW	16.9630	292.0998	28.53	-	-
	AR	8.1562	166.0216	21.44	0.4064	0.5074
	ARMA+LMS1	8.1531	165.9847	**21.21**	0.4039	0.5008
	ARMA+LMS2	7.0372	169.1651	24.44	0.3507	0.4893
	ARMA+ IA 1	9.2471	172.1396	22.59	0.4286	0.5180
	ARMA+ IA 2	**6.6153**	**162.3318**	22.51	**0.3433**	**0.4740**
	IIR+RPE	8.3423	167.0702	21.52	0.4070	0.5032
	IIR+LMS	8.3465	166.9945	21.54	0.4061	0.5036
	IIR+ IA	7.8548	159.8932	23.01	0.3897	0.4886
FURNAS 72/76	HW	26.9516	411.0062	59.47	-	-
	AR	4.9578	174.1835	21.40	0.6254	0.6513
	ARMA+LMS1	5.0954	174.5261	21.31	0.6224	0.6520
	ARMA+LMS2	4.4120	155.3570	**18.64**	0.4984	0.5777
	ARMA+ IA 1	5.0673	175.6782	21.87	0.6654	0.6671
	ARMA+ IA 2	**4.1937**	**157.6562**	19.00	**0.4970**	**0.5804**
	IIR+RPE	4.9854	175.4764	21.40	0.6174	0.6538
	IIR+LMS	4.9808	175.1214	21.38	0.6232	0.6538
	IIR+ IA	4.9565	174.7082	20.63	0.5682	0.6435
SOBRADINHO	HW	45.9117	1376.1300	32.20	-	-
	AR	40.1147	1096.1947	25.39	0.7944	0.6723
	ARMA+LMS1	34.8638	1051.5499	23.32	0.7653	0.6669
	ARMA+LMS2	**17.1287**	**764.6008**	**17.83**	**0.4650**	**0.5278**
	ARMA+ IA 1	32.9608	1029.0909	22.65	0.7551	0.6650
	ARMA+ IA 2	20.8338	839.8754	18.82	0.4910	0.5464
	IIR+RPE	37.8120	1076.3597	24.39	0.7751	0.6701
	IIR+LMS	35.4638	1057.5056	23.42	0.7694	0.6685
	IIR+ IA	35.9654	1062.4966	23.60	0.7751	0.6696
EMBORCAÇÃO	HW	6.2002	190.5383	50.64	-	-
	AR	3.5208	131.5846	25.96	0.7772	0.7025
	ARMA+LMS1	3.6168	132.1966	25.64	0.7769	0.6940
	ARMA+LMS2	2.6428	100.0613	21.63	0.6014	0.5491
	ARMA+ IA 1	3.6141	133.9609	26.59	0.8171	0.7370
	ARMA+ IA 2	**2.1581**	**94.1444**	**20.01**	**0.5393**	**0.5401**
	IIR+RPE	3.5334	132.6184	26.20	0.7816	0.7088
	IIR+LMS	3.5319	132.5575	26.20	0.7815	0.7089
	IIR+ IA	3.1094	118.5371	24.89	0.7291	0.6573

Ideally, a recurrent method may extract high generality solutions and improve the performance of a linear approach. The results showed that the insertion of a feedback response often led to the best solutions, once more information is available to the formation of the response of the predictor.

Regarding the optimization procedure, the IA was the best in 8 cases and LMS in 5. Analyzing just the optimization of the IIR filter, in 11 cases, the IAs provided better results. Also, the ARMA model presented in Equation (7) almost invariably (to h higher than 1) presented results better than those from Equation (6).

Table 6: Results for all streamflow series and six steps ahead (h = 6).

	Model	MSE real ($\times 10^4$)	MAE real	MAPE real	MSE deseas	MAE deseas
FURNAS 52/56	HW	17.5895	302.2370	29.53	-	-
	AR	6.9484	165.9277	21.35	0.4511	0.5441
	ARMA+LMS1	6.8478	164.2319	**20.76**	0.5047	0.5530
	ARMA+LMS2	6.6263	166.0349	22.54	0.3667	0.5059
	ARMA+ IA 1	6.0692	164.3675	21.64	0.4569	0.5621
	ARMA+ IA 2	5.9779	163.1211	20.78	**0.3654**	**0.5015**
	IIR+RPE	6.9733	166.2771	21.39	0.4539	0.5451
	IIR+LMS	6.4813	168.3400	23.06	0.5094	0.5852
	IIR+ IA	**5.7267**	**146.0020**	22.39	0.4346	0.5148
FURNAS 72/76	HW	25.2952	383.2954	62.63	-	-
	AR	5.7656	186.0062	23.91	0.8704	0.7275
	ARMA+LMS1	5.5854	182.7388	23.41	0.8411	0.7232
	ARMA+LMS2	4.4590	158.6120	**19.24**	0.5586	0.5993
	ARMA+ IA 1	5.9157	188.2496	24.06	0.8737	0.7340
	ARMA+ IA 2	**4.4315**	**162.4742**	20.06	**0.6261**	**0.6310**
	IIR+RPE	5.7453	185.9357	23.90	0.8742	0.7306
	IIR+LMS	5.6516	184.5136	21.92	0.7456	0.6992
	IIR+ IA	5.6938	186.0797	22.22	0.7618	0.7075
SOBRADINHO	HW	42.8919	1367.2676	32.37	-	-
	AR	24.2060	952.0566	22.99	0.9168	0.7414
	ARMA+LMS1	23.8601	952.2272	22.31	0.8985	0.7273
	ARMA+LMS2	**10.3441**	**683.1941**	**17.39**	**0.4633**	**0.5537**
	ARMA+ IA 1	24.4221	951.8605	22.66	0.9033	0.7354
	ARMA+ IA 2	12.9949	716.4950	17.82	0.5405	0.5677
	IIR+RPE	23.6769	946.7702	22.96	0.9100	0.7415
	IIR+LMS	23.9110	945.8540	22.83	0.9066	0.7366
	IIR+ IA	23.5099	938.1707	22.37	0.8870	0.7257
EMBORCAÇÃO	HW	6.8209	199.7716	58.22	-	-
	AR	3.3228	127.3918	27.78	0.8753	0.7636
	ARMA+LMS1	3.9823	140.4540	28.41	0.8910	0.7634
	ARMA+LMS2	2.4692	100.8096	23.10	0.6160	0.5874
	ARMA+ IA 1	3.3538	133.1688	30.13	0.9300	0.8143
	ARMA+ IA 2	**2.3190**	**102.6668**	**22.51**	**0.6008**	**0.6111**
	IIR+RPE	3.3354	127.5599	27.75	0.8750	0.7624
	IIR+LMS	3.3242	127.1329	27.74	0.8811	0.7614
	IIR+ IA	2.9934	117.5704	26.19	0.8013	0.7143

The empirical evidences of this study showed, in summary:

i) the presence of feedback loops (ARMA and IIR filter) improved the performances compared to the AR model to multi-step ahead prediction. In most cases, the ARMA model of Equation (7) proved to be more effective;

ii) optimization algorithms based on estimates of the gradient of the cost function are already employed. However, the use of a population metaheuristic showed very competitive, presenting good results;

iii) besides the deseasonalization of Equation (22) be the most effective until this moment, it is necessary to find alternatives that valorize the response of the predictors (which occurs in deseasonalized space) once the best response of the models may be degraded by the reversion of the process.

Table 7: Results for all streamflow series and twelve steps ahead (h = 12).

	Model	MSE real (×10^4)	MAE real	MAPE real	MSE deseas	MAE deseas
FURNAS 52/56	HW	10.6741	195.6570	25.07	-	-
	AR	10.2104	207.6676	27.17	0.6871	0.6992
	ARMA+LMS1	10.7560	199.1596	34.33	0.6544	0.6363
	ARMA+LMS2	7.4008	175.0472	21.62	**0.4337**	0.5481
	ARMA+ IA 1	10.1017	206.1241	27.17	0.6737	0.6929
	ARMA+ IA 2	**7.3119**	171.9934	**21.42**	0.4397	**0.5420**
	IIR+RPE	10.0649	206.5301	26.99	0.6798	0.6957
	IIR+LMS	10.0977	206.7539	27.02	0.6812	0.6963
	IIR+ IA	7.9468	**170.7382**	28.32	0.5810	0.5911
FURNAS 72/76	HW	14.5841	293.6045	47.55	-	-
	AR	5.4783	178.1599	22.85	0.8191	0.6897
	ARMA+LMS1	5.3730	184.8272	22.04	0.7460	0.7126
	ARMA+LMS2	4.2680	152.0157	18.09	0.5315	0.5692
	ARMA+ IA 1	5.2866	174.9972	21.92	0.7711	0.6725
	ARMA+ IA2	**3.8210**	**147.3185**	**18.07**	**0.5489**	**0.5669**
	IIR+RPE	5.5205	172.3159	21.87	0.7888	0.6722
	IIR+LMS	5.4584	177.5435	22.47	0.7873	0.6829
	IIR+ IA	5.4487	177.3626	22.48	0.7881	0.6826
SOBRADINHO	HW	47.2633	1431.4180	36.43	-	-
	AR	25.6825	1040.8319	24.29	1.0364	0.8085
	ARMA+LMS1	26.6915	1117.2437	28.47	1.2010	0.8817
	ARMA+LMS2	**10.8632**	**667.1912**	**16.78**	**0.5007**	**0.5424**
	ARMA+ IA1	26.1734	1070.4998	25.33	1.0878	0.8315
	ARMA+ IA2	11.3811	680.8712	16.82	0.5089	0.5451
	IIR+RPE	21.4678	958.0474	22.83	0.9474	0.7752
	IIR+LMS	21.1604	950.8522	22.71	0.9397	0.7721
	IIR+ IA	24.8570	1079.7776	28.32	0.9850	0.8250
EMBORCAÇÃO	HW	7.8649	201.5132	41.80	-	-
	AR	3.8799	144.4730	35.55	1.1883	0.9009
	ARMA+LMS1	4.4007	157.1772	38.56	1.3375	0.9758
	ARMA+LMS2	2.9201	112.7873	26.87	0.7861	0.6777
	ARMA+ IA 1	4.4196	141.3258	65.17	1.5693	0.9315
	ARMA+ IA 2	**2.3477**	**109.5475**	**25.84**	**0.7311**	**0.6872**
	IIR+RPE	5.4160	174.1206	47.61	1.6325	1.0846
	IIR+LMS	4.0604	149.9718	37.66	1.2585	0.9403
	IIR+ IA	4.0102	149.3458	37.57	1.2543	0.9394

As the forecasting horizon grows, the output response of the models tends to the long-term mean (LTM). We expected degradation in error with the increase of h since the correlation between the observations decreases. However, as showed in Figure 1, this correspondence was not perfect because sometimes the MSE in a higher horizon was lower than the previous.

6 Conclusions

This work proposed the application of recursive linear models - autoregressive and moving average models (ARMA) and infinite impulse response (IIR) filters - to monthly seasonal streamflow series forecasting of the following Brazilian hydroelectric plant: *Furnas*, *Sobradinho*, and *Emborcação*, with 1, 3, 6, and 12 steps ahead. This problem is fundamental for countries where the power generation is predominantly done by this kind of source.

The Brazilian Electric Sector often uses models without feedback, especially the autoregressive models (AR), since it has a closed-form solution to calculate their free parameters. Unlike the AR model, recursive models have cost functions potentially multimodal, which requires optimization procedures to find their parameters. However, such approaches raise the prospect of obtaining linear solutions with high generality, which may bring important performance gains to this problem.

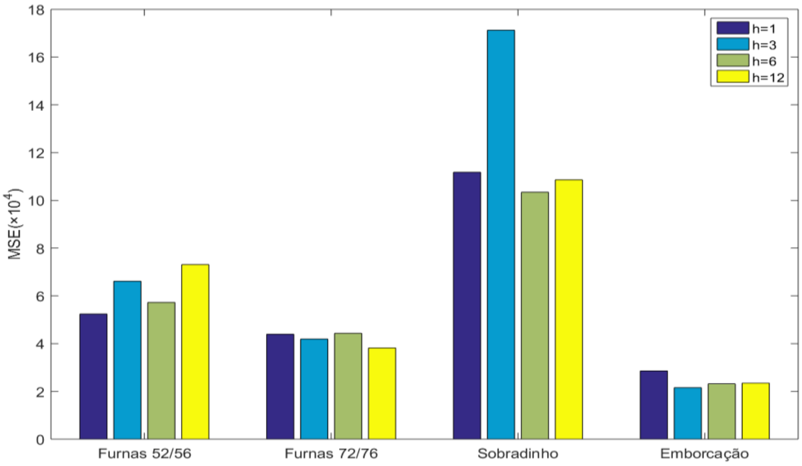

Figure 1: MSE for each forecasting horizon *h*.

Finally, Figures 2 to 5 present the streamflow series and the predicted series associated with the model that reached the best performance, considering the one step ahead prediction (h = 1).

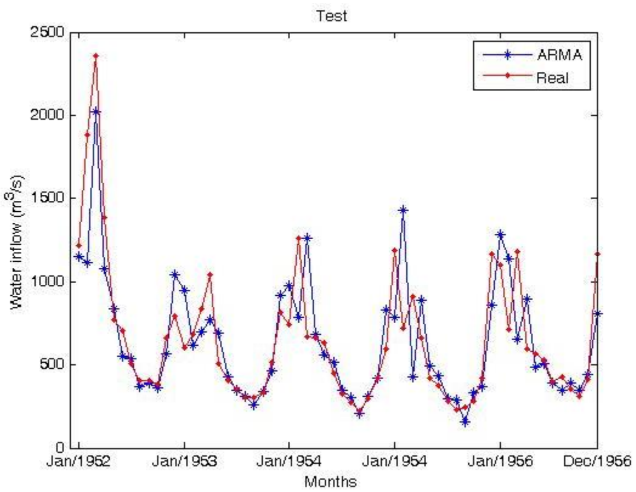

Figure 2: Best predicted series considering the test set associated with *Furnas* 1952/1956 and h = 1.

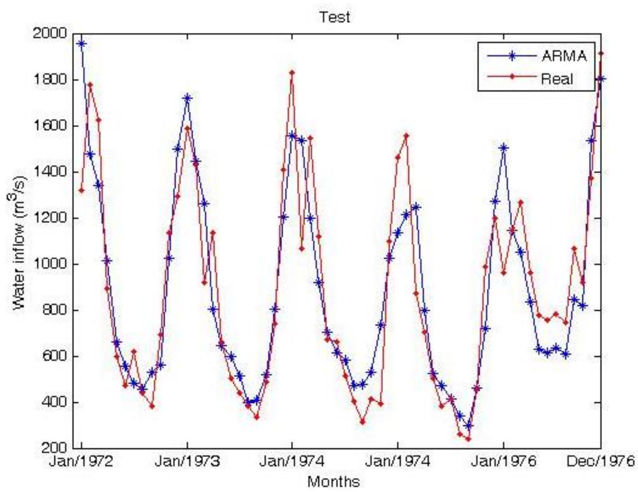

Figure 3: Best predicted series considering the test set associated with *Furnas* 1972/1976 and h = 1.

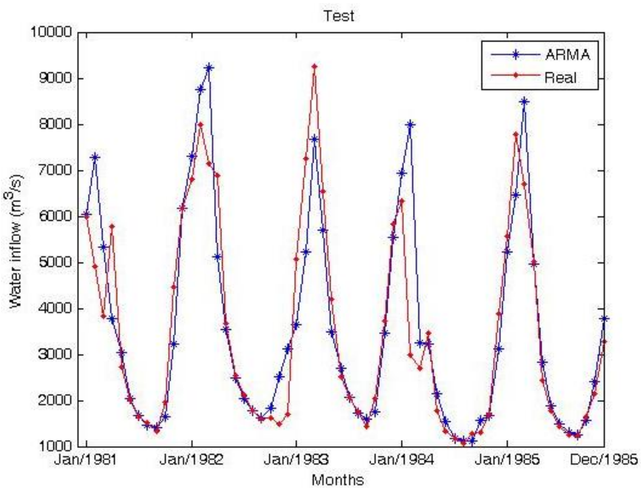

Figure 4: Best predicted series considering the test set associated with *Sobradinho* and h = 1.

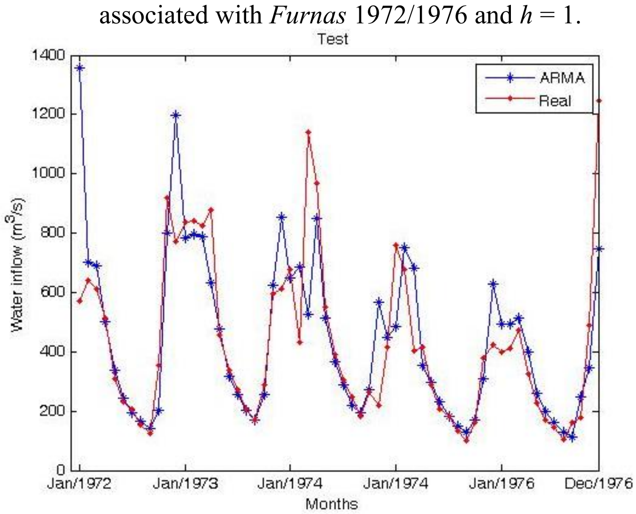

Figure 5: Best predicted series considering the test set associated with *Emborcação* and h = 1.

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An immune algorithm performs the adjustment of the ARMA and IIR filter parameters. It is a population metaheuristic inspired by the immune system of the superior organisms, which allies local search capability with high global search potential due to its intrinsic optimization mechanism based on the binomial cloning/mutation. For comparison purposes, two techniques, both elaborated based on information from the gradient: Least Mean Square (LMS) - to ARMA and IIR - and Recursive Prediction Error (RPE) - just to IIR filter. Besides, the Holt-Winter method was performed.

The results show that recurrent models bring to this problem a consistent increase in performance. Most of the best results were favorable to the ARMA model. Comparing just IIR filter and AR, once again, the recurrent approach was the best. It seems to be clear that the use of feedback is essential to linear models applied to this forecasting problem, even the loss of deterministic solutions to calculate the parameters.

Regarding the optimization algorithms, the performance achieved by the IAs was better than the others. We can state that the IAs have to be considered as a competitive candidate to solve this task.

Future works may be developed to explore this proposal using other series with hydrological behavior distinct from those. A new proposal is necessary to deseasonalize the streamflow series that maintain the proportionality between the output response of the predictor and the series in real space. In many cases, the response of the model was degraded in the reversion of the deseasonalization. In addition, a further analysis comparing the linear models and machine learning models should be evaluated (Tadano et al., 2016).

7 Acknowledgment

The authors would like to thank the National Council for Scientific and Technological Development (CNPq), processes number 432423/2016-8, 315298/2020-0, and 405580/2018-5, and Araucaria Foundation process #51497 for their financial support.

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